ANKARA YILDIRIM BEYAZIT UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

VIBRO-ACOUSTIC DESIGN, MANUFACTURING AND CHARACTERIZATION OF A TONPILZ -TYPE UNDERWATER ACOUSTIC DEVICE

M.Sc. Thesis by
Polat KURT

Department of Mechanical Engineering
June, 2017
ANKARA
VIBRO-ACOUSTIC DESIGN, MANUFACTURING AND CHARACTERIZATION OF A TONPILZ -TYPE UNDERWATER ACOUSTIC DEVICE

A Thesis Submitted to the
Graduate School of Natural and Applied Sciences of Ankara Yıldırım Beyazıt University
In Partial Fulfillment of the Requirements for the Master of Science in Mechanical Engineering, Department of Mechanical Engineering

by

Polat KURT

June, 2017

ANKARA
We have read the thesis entitled “VIBRO-ACOUSTIC DESIGN, MANUFACTURING AND CHARACTERIZATION OF A TONPILZ -TYPE UNDERWATER ACOUSTIC DEVICE” completed by POLAT KURT under the supervisions of PROF. DR. SADETTİN ORHAN and PROF. DR. CİHANGİR DURAN and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

__________________________
Prof. Dr. Sadettin ORHAN
Supervisor

__________________________
Prof. Dr. Cihangir DURAN
Co-Supervisor

__________________________
Doç. Dr. Arif ANKARALI
Jury Member

__________________________
Prof. Dr. Nizami AKTÜRK
Jury Member

__________________________
Yrd. Doç. Dr. Tuncay KARAÇAY
Jury Member

__________________________
Prof. Dr. Fatih V. ÇELEBİ
Director
Graduate School of Natural and Applied Sciences
ETHICAL DECLARATION

I hereby declare that, in this thesis which has been prepared in accordance with the Thesis Writing Manual of Graduate School of Natural and Applied Sciences,

- All data, information and documents are obtained in the framework of academic and ethical rules,

- All information, documents and assessments are presented in accordance with scientific ethics and morals,

- All the materials that have been utilized are fully cited and referenced,

- No change has been made on the utilized materials,

- All the works presented are original,

and in any contrary case of above statements, I accept to renounce all my legal rights.
ACKNOWLEDGEMENTS

Firstly, I would like to express my sincere gratitude and appreciation to my supervisors, Prof. Dr. Sadettin ORHAN and Prof. Dr. Cihangir DURAN for their tremendous support, guidance and help during my study. I feel so lucky to have the chance of conducting my studies under their supervision and support.

I am also grateful to METEKSAN Defense Industry Inc. for its support and help in the fabrication of transducers and acoustic measurements.

I would like to give my special thanks to my friends Tuba AVCI for her edits on text and Necati UÇAK for his technical supports.

My deepest thanks are for my mom and dad, Aynur and Ali KURT, for their endless support, patience and love throughout my entire life. I am also thankful to my lovely sisters Sevdenur and Beyza KURT for their emotional supports.

2017, 17 June  
Polat KURT
VIBRO-ACOUSTIC DESIGN, MANUFACTURING AND CHARACTERIZATION OF A TONPILZ-TYPE UNDERWATER ACOUSTIC DEVICE

ABSTRACT

Sonar is an electro-acoustic system that can detect an object and give information about the location of this object with the help of sound waves. The most important parts of sonar systems are transducers which are in contact with water and produce and receive sound waves. Tonpilz-type transducer is one of the most popular type of transducers used in underwater-acoustics. It is used generally for low frequency applications. This thesis covers designing, manufacturing and performing underwater acoustic tests of Tonpilz-type transducers operated below 7 kHz. Firstly, transducers are designed according to the technical specifications. Design of the transducers consists of three steps as simple lumped parameter method, equivalent circuit method and finite element method, respectively. Three transducers are specified at the end of the design process by considering the predefined specifications and then they are manufactured. Characterization and underwater acoustic tests of the transducers are performed at acoustic measurement facilities of the Meteksan Defense Industry Inc. (Ankara). When the results from the measurements are compared with the results from the design stages, it is seen that the results are matched well to each other. Maximum TVR values are found as 132.3 dB at 4.7 kHz, 133.7 dB at 4.9 kHz and 137.3 dB at 4.7 kHz for the three different manufactured transducers. Maximum RVS values are found as -154.7 dB at 4.7 kHz, -159.1 dB at 4.9 kHz and -156.8 dB at 5 kHz. As a result, three low-frequency transducers having sufficient acoustic performances have been successfully designed and manufactured.

Keywords: Tonpilz-type transducer, underwater acoustics, vibro acoustics, characterization, design, manufacturing
Sonar, bir nesneyi algılayabilen ve ses dalgalarının yardımcıla bu nesnenin konumu hakkında bilgi verebilen bir elektro-akustik sistemdir. Sonar sistemlerinin en önemli bileşenlerinden biri, suyla temas halinde olan ve ses dalgalarını üretir ve algılayan transdüserlerdir. Tonpilz tipi transdüserler su altında akustikte en çok kullanılan transdüserler tiplerinden biridir. Genellikle düşük frekanslı uygulamalar için kullanılırlar. Bu tez, 7 kHz civarında veya altında çalışan bir Tonpilz dönüştürücünün tasarlanması, üretimesi ve sualtı akustik testlerinin gerçekleştirilmesini kapsar. İlk olarak, dönüştürücüler istenilen teknik özelliklere göre tasarlanmıştır. Dönüştürücülerin tasarım sırasıyla, toplu parametre yöntemi, eşdeğer devre yöntemi ve sonlu elemanlar yöntemi olmak üzere üç basamaktan oluşur. Tasarım işleminin sonunda daha önceden belirlenen parametrelerle uygun olarak üç adet transdüser tasarlanmış ve üretilmiştir. Üretilen transdüserlerin karakterizasyonu ve su altında akustik testleri, Meteksan Defense Industry Inc (Ankara)’nin ölçüm tesislerinde gerçekleştirilmiştir. Ölçümlerden elde edilen sonuçlar tasarım aşamalarında elde edilen sonuçlarla kıyaslandığında, sonuçların büyük ölçüde eşleştiği görülmuştur. En yüksek TVR değerleri üretilen üç farklı transdüser için, 4.7 kHz’de 132.3 dB, 4.9 kHz’de 133.7 dB ve 4.7 kHz’de 137.3 dB olarak hesaplanmıştır. En yüksek RVS değerleri ise 4.7 kHz’de -154.7 dB, 4.9 kHz’de -159.1 dB ve 5 kHz’de -156.8 dB olarak bulunmuştur. Böylece, istenilen frekansta yeterli akustik performansı göstererek çalışan transdüserlerin tasarımını ve üretimi başarı ile tamamlanmıştır.

**Anahtar Kelimeler:** Tonpilz transdüser, su altında akustiği, vibroakustik, karakterizaston, tasarım, üretim
CONTENTS

M.Sc. THESIS EXAMINATION RESULT FORM ...................................................... ii
ETHICAL DECLARATION ...................................................................................... iii
ACKNOWLEDGEMENTS ....................................................................................... iv
ABSTRACT ............................................................................................................. v
ÖZ ........................................................................................................................... vi
NOMENCLATURE ................................................................................................. ix
LIST OF TABLES .................................................................................................. xiv
LIST OF FIGURES ................................................................................................ xv

CHAPTER 1 - INTRODUCTION .............................................................................. 1
  1.1 Fundamentals of Sound .............................................................................. 2
  1.2 Wave Equation and Sound Propagation from Vibrating Surfaces .............. 3
  1.3 Sonar and Underwater Transducers .......................................................... 6
  1.4 Piezoelectricity .......................................................................................... 9
  1.5 Tonpilz Type Transducers ........................................................................ 10
  1.6 Performance Parameters of Tonpilz Type Transducers ............................ 14
  1.6 Review of Related Works ......................................................................... 19

CHAPTER 2 - DESIGN METHODS AND MEASUREMENT PROCEDURES
OF TRANSDUCERS .......................................................................................... 22
  2.1 Simple Lumped Parameter Method .......................................................... 22
  2.2 Equivalent Circuit Method ........................................................................ 26
  2.3 Finite Element Method ............................................................................. 31
  2.4 Calibration and Measurement of the Transducers .................................... 35
    2.4.1 Primary Methods ............................................................................... 36
    2.4.2 Secondary Methods .......................................................................... 38
    2.4.3 Measurement Facilities .................................................................... 39

CHAPTER 3 - DESIGN OF THE TRANSDUCER ................................................ 43
  3.1 Design Parameters .................................................................................... 43
  3.2 Simple Lumped Parameter Method .......................................................... 44
  3.3 Equivalent Circuit Method ........................................................................ 47
  3.4 Finite Element Method ............................................................................. 52

CHAPTER 4 - MANUFACTURING AND CHARACTERIZATION OF THE
SELECTED TRANSDUCERS .............................................................................. 58
  4.1 Manufacturing Steps ................................................................................. 58
4.2 Characterization and Measurement

CHAPTER 5 - RESULTS AND DISCUSSION

5.1 Transducer Set-8
5.2 Transducer Set-16
5.3 Transducer Set-22

CHAPTER 6 - CONCLUSION

REFERENCES

APPENDICES

Appendix A – Radiation Impedance Calculations
Appendix B – Piezoelectric Material Properties
Appendix C – Matlab Code for Equivalent Circuit Method

CURRICULUM VITAE
NOMENCLATURE

**Roman Letter Symbols**

- $a$  Edge length, $m$
- $A$  Area, $m^2$
- $A_{pc}$  Area of piezoceramics, $m^2$
- $A_h$  Area of head mass, $m^2$
- $A_S$  Surface area of the source, $m^2$
- $A_{sr}$  Cross sectional area of stress rod, $m^2$
- $A_t$  Area of tail mass, $m^2$
- $b$  Edge length, $m$
- $B$  Susceptance, $S$
- $c$  Sound speed, $m/s$
- $c_{ij}^D$  Elastic stiffness coefficients at constant electric displacement, $N/m^2$
- $c_{ij}^E$  Elastic stiffness coefficients at constant electric field, $N/m^2$
- $C_0$  Clamped capacitance, $C/V$
- $C_{cs}$  Capacitance of piezoceramic stack, $C/V$
- $C_e$  Equivalent capacitance, $C/V$
- $C_f$  Capacitance of the transducer under free conditions, $C/V$
- $C^S$  Clamped electrical capacitance matrix, $C/V$
- $C_{sr}$  Capacitance of the stress rod, $C/V$
- $D_f$  Directivity factor
- $d_{ij}$  Piezoelectric charge constant, $C/N$
- $D_i$  Electric displacement, $C/m^2$
- $d$  Diameter, $m$
- $d_o$  Outer diameter of tail mass, $m$
- $d_i$  Inner diameter of tail mass, $m$
- $d_{sr}$  Diameter of stress rod, $m$
- $E_i$  Electric field, $V/m$
- $e_{ij}$  Piezoelectric constant, $C/m^2$
- $f$  Frequency, $1/s$
\( F \) Force, \( N \)
\( f_1, f_2 \) Frequencies of 3-dB below maximum, \( 1/s \)
\( F_b \) Blocking force, \( N \)
\( F_e \) Excitation force, \( N \)
\( f_{flex} \) Flexural frequency, \( 1/s \)
\( f_n \) Natural frequency, \( 1/s \)
\( f_p \) Peak frequency, \( 1/s \)
\( F_t \) Total force, \( N \)
\( G \) Conductance, \( S \)
\( g_{ij} \) Piezoelectric voltage constant, \( Vm/N \)
\( h_{ij} \) Piezoelectric constant, \( V/m \)
\( i \) Current, \( A \)
\( i_p \) Input current of projector, \( A \)
\( i_{PH} \) Input voltage for projector hydrophone pair, \( A \)
\( i_{PR} \) Input voltage for projector reciprocal transducer pair, \( A \)
\( i_{RH} \) Input voltage for reciprocal transducer hydrophone pair, \( A \)
\( i_{RP} \) Input voltage for reciprocal transducer projector pair, \( A \)
\( I \) Intensity, \( W/m^2 \)
\( I_a \) The average intensity in all directions, \( W/m^2 \)
\( I_0 \) Acoustic intensity in the acoustic axis, \( W/m^2 \)
\( J \) Reciprocity parameter
\( K^E \) Stiffness matrix, \( N/m \)
\( K_c \) Short circuit stiffness, \( N/m \)
\( K_e \) Effective stiffness, \( N/m \)
\( K_g \) Stiffness of glue, \( N/m \)
\( K_{cs} \) Stiffness of the piezoceramic stack, \( N/m \)
\( K_{sr} \) Stiffness of stress rod, \( N/m \)
\( k_{ij} \) Electromechanical coupling factor
\( k \) Wave number, \( rad/m \)
\( l \) Length, \( m \)
\( l_{cs} \) Length of piezoceramic stack, \( m \)
\( l_{pc} \) Length of piezoceramic, \( m \)
\( l_t \) Length of tail mass, \( m \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{sr}$</td>
<td>Length of stress rod, m</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass, kg</td>
</tr>
<tr>
<td>$M_{cs}$</td>
<td>Mass of piezoceramic stack, kg</td>
</tr>
<tr>
<td>$M_e$</td>
<td>Effective mass, kg</td>
</tr>
<tr>
<td>$M_h$</td>
<td>Mass of head mass, kg</td>
</tr>
<tr>
<td>$M_t$</td>
<td>Mass of tail mass, kg</td>
</tr>
<tr>
<td>$N$</td>
<td>Transduction coefficient, $N/V$</td>
</tr>
<tr>
<td>$n_{pc}$</td>
<td>Number of piezoceramics in a stack</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure, Pa</td>
</tr>
<tr>
<td>$P_{ff}$</td>
<td>Free field acoustic pressure, Pa</td>
</tr>
<tr>
<td>$P_{ref}$</td>
<td>Reference acoustic pressure, Pa</td>
</tr>
<tr>
<td>$P_{rms}$</td>
<td>Root mean square acoustic pressure, Pa</td>
</tr>
<tr>
<td>$Q$</td>
<td>Charge vector, C</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>Mechanical quality factor</td>
</tr>
<tr>
<td>$r$</td>
<td>Distance, m</td>
</tr>
<tr>
<td>$R$</td>
<td>Electrical resistance, $\Omega$</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Reference far field distance, m</td>
</tr>
<tr>
<td>$R_0$</td>
<td>Electrical resistance of piezoceramics, $\Omega$</td>
</tr>
<tr>
<td>$R_e$</td>
<td>Effective resistance, $\Omega$</td>
</tr>
<tr>
<td>$r_{ff}$</td>
<td>Far field distance, m</td>
</tr>
<tr>
<td>$R_m$</td>
<td>Mechanical resistance due to mechanical loss, $\Omega$</td>
</tr>
<tr>
<td>$R_r$</td>
<td>Radiation resistance,</td>
</tr>
<tr>
<td>$r_{pc,o}$</td>
<td>Outer radius of piezoceramic, m</td>
</tr>
<tr>
<td>$r_{pc,i}$</td>
<td>Inner radius of piezoceramic, m</td>
</tr>
<tr>
<td>$r_{t,o}$</td>
<td>Outer radius of tail mass, m</td>
</tr>
<tr>
<td>$r_{sr}$</td>
<td>Radius of stress rod, m</td>
</tr>
<tr>
<td>$S$</td>
<td>Transmitting voltage response, $Pa/V$</td>
</tr>
<tr>
<td>$s_{ij}^D$</td>
<td>Elastic compliance at constant electric displacement, $m^2/N$</td>
</tr>
<tr>
<td>$s_{ij}^E$</td>
<td>Elastic compliance at constant electric field, $m^2/N$</td>
</tr>
<tr>
<td>$T$</td>
<td>Period</td>
</tr>
<tr>
<td>$t$</td>
<td>Time, t</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Stress, $N/m^2$</td>
</tr>
<tr>
<td>$t_h$</td>
<td>Thickness of head mass, m</td>
</tr>
</tbody>
</table>


\( t_{pc} \)  Thickness of piezoceramics, \( m \)
\( t_{\text{flex}} \)  Minimum thickness regarding resonance frequency of head mass, \( m \)
\( \tan \delta \)  Electrical dissipation factor
\( u \)  Velocity of the vibrating source, \( m/s \)
\( u_h \)  Velocity of head mass, \( m/s \)
\( u_r \)  Relative velocity, \( m/s \)
\( u_t \)  Velocity of tail mass, \( m/s \)
\( V \)  Voltage, \( V \)
\( V_H \)  Open-circuit voltage of hydrophone, \( V \)
\( V_{PH} \)  Output voltage for projector hydrophone pair, \( V \)
\( V_{PR} \)  Output voltage for projector reciprocal transducer pair, \( V \)
\( V_{RH} \)  Output voltage for reciprocal transducer hydrophone pair, \( V \)
\( V_{RP} \)  Output voltage for reciprocal transducer projector pair, \( V \)
\( V_{drive} \)  Drive voltage, \( V \)
\( W \)  Sound power, \( W \)
\( W_e \)  Input electrical power, \( W \)
\( Y_a \)  Admittance, \( S \)
\( Y \)  Modulus of Elasticity, \( N/m^2 \)
\( Y_{pc} \)  Modulus of Elasticity of ceramics, \( N/m^2 \)
\( Y_{sr} \)  Modulus of Elasticity of stress rod, \( N/m^2 \)
\( Z_{PH} \)  Transfer impedance between projector and hydrophone
\( Z_{PR} \)  Transfer impedance between projector and reciprocal transducer
\( Z_{RH} \)  Transfer impedance between reciprocal transducer and hydrophone
\( Z_m \)  Mechanical impedance, \( Ns/m \)
\( x \)  Displacement of equivalent mass, \( m \)
\( x_h \)  Displacement of head mass, \( m \)
\( x_t \)  Displacement of tail mass, \( m \)

**Greek Letter Symbols**

\( \eta_{ea} \)  Electroacoustic efficiency
\( \eta_{ma} \)  Mechanoacoustic efficiency
\( \varepsilon^\sigma_{mk} \)  Permittivity coefficients at constant stress, \( F/m \)
\( \varepsilon^\varepsilon_{mk} \)  Permittivity coefficients at constant strain, \( F/m \)
\( \beta^\sigma_{mk} \)  Impermittivity coefficients at constant stress, \( m/F \)
Impermittivity coefficients at constant strain, $m/F$

Strain

Tail to head mass ratio

Density, $kg/m^3$

Density of head mass, $kg/m^3$

Density of piezoceramics, $kg/m^3$

Density of tail mass, $kg/m^3$

Natural frequency, $rad/s$

Spherical angle, $rad$

Spherical coordinate

Change in thickness of ceramics due to applied electric field, $m$

Wavelength, $m$

Angular freq, $rad/s$

Poisson ratio

Free field sensitivity of hydrophone, $V/Pa$

Free field sensitivity of reciprocal transducer, $V/Pa$

Receiving sensitivity, $V/Pa$

Free field sensitivity of standard hydrophone, $V/Pa$

Free field sensitivity of the unknown hydrophone, $V/Pa$

**Acronyms**

DI Directivity index

FEA Finite Element Analysis

FEM Finite Element Method

RVS Receiving Voltage Sensitivity

SL Source Level

SPL Sound Pressure Level

TVR Transmitting Voltage Response
LIST OF TABLES

Table 2.1 Electrical equivalence of the mechanical parts ........................................ 26
Table 3.1 Dimensions of parts obtained in first step .................................................. 46
Table 3.2 Parameters obtained from ECM ................................................................. 49
Table 3.3 Parameters obtained from ECM (improved) ............................................... 50
Table 3.4 Results obtained with Finite Element Method ............................................ 55
Table 5.1 Comparison of the resonance frequencies .................................................... 67
Table 5.2 Comparison of the TVR results ................................................................. 68
Table 5.3 Comparison of the peak RVS frequency and the RVS results ....................... 69
Table 5.4 Comparison of the results ........................................................................... 70
Table 5.5 Comparison of the resonance frequencies .................................................... 71
Table 5.6 Comparison of the TVR results ................................................................. 72
Table 5.7 Comparison of the peak RVS frequency and the RVS results ....................... 73
Table 5.8 Comparison of the results ........................................................................... 74
Table 5.9 Comparison of the resonance frequencies .................................................... 75
Table 5.10 Comparison of the TVR results ............................................................... 76
Table 5.11 Comparison of the peak RVS frequency and the RVS results ....................... 77
Table 5.12 Comparison of the results ........................................................................... 78
Table 5.13 Comparison of constraints of the models and the actual measurement ... 82
Table 5.14 The relative deviations (%) calculated in the design methods ................. 83
LIST OF FIGURES

Figure 1.1 Working principle of active sonars .......................................................... 7
Figure 1.2 Polarization of piezoelectric ceramics ....................................................... 10
Figure 1.3 Piezoelectric effects, generator and motor actions ................................. 10
Figure 1.4 Photograph of transducer examples ......................................................... 11
Figure 1.5 Photograph of a Tonpilz-type transducer .................................................. 11
Figure 1.6 Cross sectional view of a Tonpilz transducer ........................................... 13
Figure 1.7 Admittance of a tonpilz-transducer ........................................................ 14
Figure 1.8 Bandwidth calculation from a TVR plot .................................................... 17
Figure 1.9 Schematic description of a transducer beam pattern .................................. 18
Figure 2.1 Mass spring system representation of a transducer ................................... 22
Figure 2.2 Electrical equivalent circuit of the system shown in Figure 2.1 ............... 23
Figure 2.3 SDF mass-spring system ........................................................................... 23
Figure 2.4 Simple model of a Tonpilz transducer ...................................................... 27
Figure 2.5 Electrical equivalent circuit ....................................................................... 27
Figure 2.6 Quarter section of Tonpilz transducer in COMSOL ................................. 34
Figure 2.7 Applied voltage and polarization directions ............................................. 35
Figure 2.8 Measurements for conventional reciprocity method .............................. 37
Figure 2.9 Main sources of reflections in a measurement in water tank ................. 40
Figure 2.10 Measured data for a transducer (ITC1001) ............................................. 40
Figure 3.1 Simple schematic model of a Tonpilz transducer ..................................... 46
Figure 3.2 Equivalent circuit of the transducer ......................................................... 48
Figure 3.3 Equivalent circuit of the transducer ........................................................ 48
Figure 3.4 TVR results calculated by ECM ............................................................... 51
Figure 3.5 Cross sectional view of transducer obtained in the second step ............... 51
Figure 3.6 3D model of the transducer constructed with Solidworks ...................... 52
Figure 3.7 Model of tonpilz transducer in COMSOL ................................................. 53
Figure 3.8 Electrical potential distribution in piezoceramics .................................. 54
Figure 3.9 Mesh of the transducer and water medium .............................................. 54
Figure 3.10 Conductance results of the transducers calculated by FEM ............... 56
Figure 3.11 TVR results of the transducers calculated by FEM ............................... 56
Figure 3.12 RVS calculated by FEM .......................................................................... 57
CHAPTER 1

INTRODUCTION

The communication in water is provided by acoustic waves unlike in air because the electromagnetic waves can not propagate to large distances in water. Sonar which is acronym for ‘Sound Navigation and Ranging’ is used to define underwater acoustic technologies as communication, detection and navigation in water. Sonar can be separated into two types according to working principle as active and passive sonar. Active sonar both emits the sound waves and listens to them while passive sonar only listens to the sound waves generated by other objects. In both systems, transducers, which are defined in general meaning as a device converting one form of energy to another form, are used [1].

Transducer is the main part of a sonar system determining its characteristics and performance. They can be used as hydrophones to listen to the water or as projectors to emit the sound to the water. Transducers can be divided into groups according to their shapes and working principals. Tonpilz-type transducers are the most common type of transducers using in low-frequency applications. A Tonpilz-type transducer comprises of a head mass to transmit the sound to the water, a heavy tail mass to obtain higher powers, a piezoceramic stack to drive the system between head and tail and a bolt to hold and tighten the parts [1].

It is important to design transducers in low frequencies to reach farther distances in water. In this thesis, the design, manufacturing and characterization of Tonpilz transducers operating below 7 kHz with suitable acoustic parameters are performed. First, a brief introduction to the underwater acoustics and sonar systems are given in Chapter 1. The piezoelectricity, piezoelectric transducers and their performance parameters are also given in detail. Then, the design methods such as simple lumped parameter method, equivalent circuit method (ECM) and finite element method (FEM), are described and experimental set-up and analysis principles are stated in Chapter 2. In Chapter 3, the design criteria are specified, three transducers with low frequency and good acoustic performance are designed with SOLIDWORKS and
analyzed with COMSOL. Acoustic measurements are carried out in the pressure tank and acoustic test pool at METEKSAN Defense Industry infrastructure. The results calculated from design methods are given in detail and compared to the experimental measurements in Chapter 5. The last Chapter includes the conclusion of the study and advices for the future works.

### 1.1 Fundamentals of Sound

Sound waves can be described as sine waves in a simplest manner. The basic concepts of sound waves are frequency (f), wavelength (λ) and pressure amplitude. Frequency of a sound wave can be described as the number of waves that pass from a fixed point in a unit time. Angular frequency is alternative unit for frequency and is defined by radians per second. It has a relation with ordinary frequency as follows:

\[ \omega = 2\pi f = \frac{2\pi}{T} \]  

where \( T \) is the period of the wave and it can be defined as amount of time for a single cycle and is reverse of the frequency. Wavelength is the distance from one point in a wave to the corresponding point in another cycle of the wave. Wavelength depends on the speed of sound in the medium that the wave passes through and frequency of the wave.

Speed of sound is a characteristic property of a medium. It depends on the density and modulus of elasticity of the medium where sound travels. Sonic speed is approximately 343 m/s in air at normal conditions and 1500 m/s in water. Sound speed \( c \) for solids can be formulated as follows:

\[ c = \sqrt{\frac{Y}{\rho}} \]  

where \( Y \) represents the Young’s Modulus and \( \rho \) is density of medium. By knowing the sound speed, wavelength of pressure wave can be described as:
Another concept using in underwater acoustics is acoustic intensity \( I \). It is defined as the average rate of flow of energy through a unit area normal to the direction of wave propagation and expressed as Watts per square meter \([2]\):

\[
I = \frac{P_{\text{rms}}^2}{\rho c} = \frac{W}{A(r)}
\]  

where \( W \) is sound power and \( A(r) \) is unit area in the direction of wave propagation. The term \( \rho c \) is called as specific acoustic impedance of the medium. Acoustic impedance is an important concept in the transducer design. It characterizes the ratio between particle velocity and resulting pressure. The water has an acoustic impedance much higher than air. Therefore, water has more effect than air on the operation of the transducer and this makes underwater calculations more complicated \([1]\). The pressure amplitude obtained from the displacement amplitude of the vibrating source in underwater is much higher than that of obtained in air for same amount of displacement \([3]\).

Amplitude of sound in underwater acoustics is defined as Sound Pressure Level (SPL) which is proportion of pressure magnitude of the waves to the average local pressure in the medium and expressed in decibel (dB). SPL can be given as:

\[
SPL = 10 \log \left( \frac{P_{\text{rms}}^2}{P_{\text{ref}}^2} \right) = 20 \log \left( \frac{P_{\text{rms}}}{P_{\text{ref}}} \right)
\]  

\( P_{\text{rms}} \) is root mean square value of pressure magnitude of the sound wave. \( P_{\text{ref}} \) is reference pressure and is 20 \( \mu Pa \) for air and 1 \( \mu Pa \) for water which are lowest hearable pressure levels to human air \([4]\).

### 1.2 Wave Equation and Sound Propagation from Vibrating Surfaces

Sonar transducers generate sound in the medium through their vibrating surfaces. The acoustic medium, where transducer works, is important to determine the
characteristics of a transducer. Therefore, acoustic medium and sound propagation in that medium are significant parts of the sonar transducer design.

Transducers project sound in a directional manner which changes with distance and frequency. Sound parameters and transducer characteristics are generally calculated in far field region where the directional parameters are no more a function of distance. In the far field region, sound pressure becomes inversely proportional to the distance [1].

Sound can be described as pressure fluctuations in an elastic medium such as air or water, as a result of the vibrational forces acting on the medium. Sound has the characteristics of wave phenomenon as it is in light and radio signals, however it cannot travel through a vacuum [4].

Propagation of sound waves can be described by acoustic wave equation derived from Euler’s conservation of mass and momentum equations. The linear conservation equations can be written as [5]:

\[
\frac{\partial \bar{u}}{\partial t} + \nabla \bar{P} = 0 \tag{1.6}
\]

\[
\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \bar{u} = 0 \tag{1.7}
\]

where, \(t\) is time, \(\rho\) is acoustic density and \(P\) is acoustic pressure. The first equation is equation of motion of a particle in the medium and the second one is continuity equation. By knowing the relation between the acoustic pressure and density, \(P = c^2 \rho\), the wave equation for the acoustic pressure can be found by substituting the equation of motion to the time derivative of the continuity equation, as follows [1]:

\[
\Delta P - \frac{1}{c^2} \frac{\partial^2 P}{\partial t^2} = 0 \tag{1.8}
\]

where, \(\Delta\) is Laplacian operator and \(c\) is the sound speed in the medium.

For sinusoidal wave travelling in the medium (i.e. the pressure in the form \(Pe^{-j\omega t}\)), the Helmholtz differential equation is obtained.
\[ \Delta P + k^2 P = 0 \]  \hspace{1cm} (1.9)

In this equation, \( k = \omega/c \) is known as wave number.

Because the specific cases are considered in the thesis, the far field pressure in a distance \( r \) from a vibrating planar source is given by Rayleigh’s integral [6]:

\[ P = -j \rho c k e^{-j\omega t} \frac{1}{2\pi r} \int_{A_S} u e^{jkr} dA_S \]  \hspace{1cm} (1.10)

where, \( A_S \) is surface area of the source and \( u \) is the velocity distribution of the source. For rectangular sources with the edge lengths \( a \) and \( b \), the far field pressure distributions can be found by integrating Eq. 1.10 with the limits \(-a/2\) to \(a/2\) in \( x \) coordinate and \(b/2\) to \(b/2\) in \( y \) coordinate as [6]:

\[ P(r, \varphi_1, \varphi_2, t) \]

\[ = -j \rho c k u a b e^{i(-\omega t + kr)} \left\{ \sin\left(\frac{ka \varphi_1}{2}\right) \sin\left(\frac{kb \varphi_2}{2}\right) \right\} \]  \hspace{1cm} (1.11)

\[ \varphi_1 = \sin \theta \cos \varphi \]  \hspace{1cm} (1.12)

\[ \varphi_2 = \sin \theta \sin \varphi \]  \hspace{1cm} (1.13)

Here, the spherical coordinates \( r, \varphi \) and \( \theta \) define the location of the point where the pressure will be calculated. The amplitude of the pressure in the far field without time dependence can be found as [6]:

\[ |P(r, \varphi_1, \varphi_2)| = \frac{\rho c k u a b}{2\pi r} \left\{ \sin\left(\frac{ka \varphi_1}{2}\right) \sin\left(\frac{kb \varphi_2}{2}\right) \right\} \]  \hspace{1cm} (1.14)

Substituting the Eq. 1.14 into the Eq. 1.4 the sound intensity is obtained as follows:
\[ I(r, \varnothing, \theta) = \frac{\rho ck^2 u^2 a^2 b^2}{8\pi^2 r^2} \left( \sin \left( \frac{k a \varnothing_1}{2} \right) \sin \left( \frac{k b \varnothing_2}{2} \right) \right)^2 \]  

(1.15)

The acoustic axis is defined as the axis where maximum acoustic intensity occurs. In the far field region, the maximum acoustic intensity is found in the direction of the source, in other words when \( \varnothing = \theta = 0 \). The part in the braces converges to 1 in that case and the Eqs. 1.14 and 1.15 become:

\[ |P(r,0,0)| = \frac{\rho ck u ab}{2\pi r} \]  

(1.16)

\[ I(r,0,0) = \frac{\rho ck^2 u^2 a^2 b^2}{8\pi^2 r^2} \]  

(1.17)

The total acoustic power radiated from the source can be found with the equation below [6]:

\[ W = \int_{\theta=0}^{\pi/2} \int_{\varnothing=0}^{2\pi} I(r, \varnothing, \theta) \sin \theta \ r^2 \ d\varnothing d\theta \]  

(1.18)

1.3 Sonar and Underwater Transducers

Sonar is a technique to detect objects in underwater (mostly but not limited) by using sound waves which are absorbed and propagated in water with much lesser attenuation compared to other radiation techniques [4]. It is different from radars by means of transfer method of observed energy. Unlike the radar systems working with electromagnetic waves, sonar systems transfer the energy to the medium by mechanical vibrations [7].

The main operation of sonar is to propagate waves between target and receiver. Sonar can be divided into two types based on the working principle as active and passive sonars. Active sonar emits wave into the water and listens for the returning echo from the target objects while passive sonar only listens to the sound emitted from other objects without emitting sound [7].
Active sonar uses a sound transmitter to emit sound and a receiver to listen to water. Figure 1.1 shows the working principle of an active sonar. The wave is sent by a transmitter and the wave reflected from target object is listened by a receiver.

![Figure 1.1 Working principle of active sonars](image)

Measuring the distance of the target is based on a basic principle that the system measures the time between pulse and echo of the sound and calculates the distance with a simple formula as follows:

\[
Distance = \frac{Speed\ of\ Sound \times Elapsed\ Time}{2} \tag{1.19}
\]

Passive sonar systems are used to detect sounds emitted from other objects such as marine ships and marine animals. Since they do not send their own signal to the environment, they are useful in military applications where it is important not to be detected by enemies’ sonar or in scientific applications requiring silence while listening to the marine animals.

Both types of sonar systems need transducers to produce and listen to the sound. Devices used in the sound generation are called as projectors and devices used for detection of projected sound are called as hydrophones. Projectors are used only in active sonar systems while hydrophones can be used in active or passive sonar systems.
Transducers in active sonar systems can be used as both projector and hydrophone or sometimes it could be needed to use hydrophones and projectors separately. The use of projectors and hydrophones depends on the purpose of the application area. For example, submarines require projectors and hydrophones for communication in underwater to emit and listen the sound while some ships require only hydrophone for passive listening [1].

The range of frequencies for the applications of underwater sound is nearly from 1 Hz to over 1 MHz [1]. The working frequency of the transducer is up to requirements of the application. In water, absorption of sound increases with the increase of frequency. Therefore, low frequency transducers are used in order to reach farther distances.

Transducers are widely used in but not limited to naval applications. Mine-hunting, mine sweeping, acoustic communication, active search and depth sounding are the application areas of transducers in surface ships. Submarines depend more on sonar systems because they generally operate in underwater. Bottom mapping, fish finding, oil exploration, position marking and various research projects are other application areas of the sonar transducers [1,8].

All the application areas require specific characteristics and variety of the transducers. Therefore, many types of transducers according to their shapes or working mechanisms are designed to meet different requirements such as operating frequency, acoustic power, size, wavelength etc. Electroacoustic, electromagnetic, electrochemical and electromechanical are the transducer types on the basis of working principle. Electroacoustic transducers which basically transform the electrical signal to the sound wave are the most common types in underwater applications. Electroacoustic transducers used as projectors can be grouped according to their shapes as ring and spherical, piston, transmission line, fлектensional and flexural transducers. Hydrophones can also be divided into groups as cylindrical, spherical, planar, bender and vector.

Electroacoustic transducers operate using one of the following transduction mechanisms; piezoelectricity, electrostriction, magnetostriction, electrostatic, moving
coil and variable reluctance [1]. Since piezoelectric transducers are used in this thesis, the other types are not mentioned in detail.

1.4 Piezoelectricity

Piezoelectricity comes from Greek word ‘piezein’ which means press and electron, and was discovered by Jacques Curie and Pierre Curie in 1880. Piezoelectricity simply defined as electricity production by pressure. Piezoelectricity is a reversible process and these processes are named as direct effect and inverse (converse, indirect) effect. Direct effect refers to a linear relationship between applied mechanical stress and resulting electric polarization while inverse effect refers to the same relation between applied electric field and resulting mechanical strain. However, inverse effect of piezoelectricity could not be predicted by Curie brothers in the first discovery. This effect was then mathematically figured out by Lippmann using thermodynamic arguments in 1881 and this was confirmed by Curie brothers in next publications [9,10].

Natural materials which have piezoelectric capabilities such as quartz and tourmaline were commonly used in the sonar applications until the man-made piezoelectric ceramics also called piezoceramics discovered in the 1960s. The first man-made piezoceramic applications are made of barium titanate (BaTiO$_3$) compositions. Then, PZT (Pb(Zr,Ti)O$_3$) materials which are based on lead-zirconate-titanate materials started to dominate the sonar applications due to their high sensitivity and operating temperatures [11,12]. Compared to the natural piezoelectric materials, piezoceramics are widely used in underwater applications because of their ease of manufacturing, inexpensiveness and flexibility in the dimensions during manufacturing. They are also physically strong and chemically inert [11].

Domains in piezoelectric materials are randomly oriented but they are polarized in the same direction by applying a strong, direct current (DC). After electric field is removed, piezoelectric materials remain poled and domains are permanently elongated in the direction of the field. Fig. 1.2 shows the polarization process of the piezoceramics.
As shown in Figure 1.3, in the motor action, when a field parallel to the poling direction applied to the material, ceramic will elongate and in the case of opposite direction of the field it will become shorter. If the field is alternating current (AC), it will lengthen and shorten periodically, causing vibration at a certain frequency. On the contrary to motor action, in generator action, voltage is generated as the result of the applied stress [1,11].

**1.5 Tonpilz Type Transducers**

Piezoelectric transducers use piezoelectricity to create sound. They could have miscellaneous geometrical and mechanical shapes such as spherical, cylindrical, piston, bender and flextensional transducers as mentioned earlier. The most important
property when choosing transducer type is the operating frequency. At low frequencies, benders or flexextensional transducers are usually used while at high frequencies small transducers such as metal sandwich types are used. Transducers for naval applications are needed to operate at low frequencies and should produce high powers to reach farther distances. Among all types of transducers, Tonpilz-type transducers are the most common type for low frequency applications [1]. Figure 1.4 and Figure 1.5 show examples for some transducer types.

Figure 1.4 Photograph of transducer examples; flexextensional transducer (left) and ring transducer (right) [13]

Figure 1.5 Photograph of a Tonpilz-type transducer [14]
A Tonpilz-type transducer basically comprises of piezoceramics sandwiched with a bolt between a head mass and a comparatively heavy tail mass. The main parts of a Tonpilz transducer are head mass, tail mass, stress rod and piezoceramics. Acoustical power obtained from the transducer is substantially related to the volume of piezoceramics. They are the most important parts of the transducers. 33-mode driven piezoceramics are used in Tonpilz-type transducers [1]. The poling directions of the consecutive ceramics should have opposite directions for electrically parallel wiring in a stack. Piezoceramics should be used under compressive stresses due to their unreliability under tensile stresses [9].

The main objective of the tail mass to be a counter weight to the head mass. Because increasing the mass of the tail increases the acoustic power and bandwidth of the transducer, it is needed to be as heavy as possible. Therefore, its material is chosen generally as steel or, in some special applications, as tungsten [12].

Stress rod holds the transducer parts together and provide steady compressive stress on piezoceramics. Because it can degrade the motion of the transducer, it should be made from materials which have low stiffness. Stress rod is commonly made from high strength steel.

Head mass is the generator part of a transducer. The surface, omitting the rubber root, in contact with the water is called active surface. In order to produce high acoustic power, displacement response of the active surface should be maximized. Thus, it is preferable to increase the area of active surface. On the other hand, increasing the area can lower the flexural frequency of the head mass which should be avoided and adjusted significantly higher than operating frequency. Head mass should be stiff and light to improve radiating sound power and it is generally made from aluminum [1,12].

Also, head mass influences sound pressure level. Ratio of the head mass to the tail mass is a concept in transducer design and directly effects the sound pressure radiated by transducer. When tail mass infinitely large from head mass, $SPL$ is $6 \, dB$ higher than in the case of head mass equal to tail mass. Typically, the ratio of tail mass to head mass is chosen between 2 and 4 [1].
The other parts such as insulators which are used to protect the transducer parts from electricity in the piezoceramics, glue used to stick the transducer parts together and electrodes used for application of voltage can be defined as passive parts of a transducer. They are used not to adjust the characteristics of the transducer; however, they influence the performance parameters considerably.

Isolators are used to isolate the other parts from the vibration of the head mass. Insulators protect the head and tail masses from electrical conduction. Transformer and pig tail are the parts which provide electrical connection of the transducer [12]. Cross sectional view and description of parts of a Tonpilz transducer are shown in Figure 1.6.

![Cross sectional view of a Tonpilz transducer](image)

**Figure 1.6** Cross sectional view of a Tonpilz transducer [1]

Tonpilz transducers project sound from vibrating head mass. When an alternative current applied to the piezoceramics, transducer will start vibrating at a frequency of an applied voltage. As a result, head mass will produce sound in that frequency by means of acoustic waves. Because it is needed to produce as high acoustic power as possible, it is preferable to run the transducer at resonance frequency. While, generally, it is not preferred for mechanical parts to operate at resonance frequencies in order to prevent high vibration amplitudes, transducers are operated near resonance frequencies to have higher vibration amplitudes [12].
1.6 Performance Parameters of Tonpilz Type Transducers

Transducers operate at or near their resonance frequencies, so resonance frequency is the most important performance metric of transducers. It is a common method to use conductance or admittance in order to find resonance frequencies. Conductance or admittance are the most reliable indicators of resonance frequencies of a transducer. Admittance can be defined as ease of flow of alternative current through a complex circuit. It is the ratio of current to voltage in the alternative current circuits and conductance is the real part of admittance. They can be formulated as:

\[ i = \frac{V}{Z} = VY_a = V(G + jB) \]  

where \( i \) is current, \( V \) is voltage, \( Z \) is impedance, \( Y_a \) is admittance, that is reciprocal of impedance, \( G \) is conductance and \( B \) is susceptance. Peak values for admittance or conductance represent the resonance frequencies of the transducers. An example of admittance vs frequency graph of a transducer with a resonance frequency at 6.2 kHz is shown in Figure 1.7.

![Admittance of a tonpilz-transducer](image)
Sound pressure level (SPL) is a logarithmic scale of sound pressure according to a reference value which is 20 $\mu$Pa for air and 1 $\mu$Pa for water. Transmitting voltage response (TVR) and source level (SL) are generally used as indicators of sound pressure level in transducers. They are defined as the ratio of acoustic pressure generated at 1 m distance in acoustic axis of a transducer to a reference pressure. The only difference between them is that TVR is calculated for a driving voltage of 1 Volt while it is not limited for SL [1,12]. TVR can be formulated as:

$$TVR = 10 \log \left( \frac{\rho c W_e}{4 \pi (1 \mu Pa)^2} \right) + DI + 10 \log(\eta_{ea}) \text{ dB}$$ (1.21)

$$TVR = 10 \log(W_e) + DI + 10 \log(\eta_{ea}) + 170.8 \text{ dB}$$ (1.22)

where $DI$ is directivity index of the transducer, $W_e$ is the input electrical power and calculated for 1 Volt driving voltage and $\eta_{ea}$ is electroacoustic efficiency. This equation is for distance of 1 m and for water medium. Therefore, \( \rho c = 1.5 \times 10^6 \text{ kg/m}^2\text{s} \) and reference pressure is 1 $\mu$Pa. For different drive voltages, source level can be calculated as,

$$SL = TVR + 20 \log \left( \frac{V_{drive}}{1 \text{ Volt}} \right)$$ (1.23)

Directivity of a transducer is defined as the variation of acoustic intensity with the angles $\theta$ and $\phi$ in a distance in the far field. Directivity factor is the ratio of acoustic intensity in the acoustic axis, $I_0(r)$, to the average intensity in all directions, $I_a$, in the far field distance, $r$. They can be formulated as:

$$I_a(r) = \frac{W}{4\pi r^2}$$ (1.24)

$$D_f = \frac{I_0(r)}{I_a(r)} = \frac{I_0(r)}{W/4\pi r^2}$$ (1.25)

Directivity index is the decibel scale ratio of directivity factor,
\[ DI = 10 \log(D_f) \]  

Receiving sensitivity is another parameter used to calculate acoustic performance of the hydrophones. It is the ratio of the open circuit voltage to the free field pressure which is the pressure at the hydrophone location.

\[ \mathcal{M} = \frac{V}{P_{ff}} \]  

(1.27)

Receiving voltage sensitivity (RVS) or free field voltage sensitivity are used as dB scale ratio of the receiving sensitivity. Transducer may be analyzed as projector and then RVS can be obtained also with the reciprocity formula as follows:

\[ RVS = 20 \log(\mathcal{M}) = TVR + 20 \log|Z| - 20 \log(f) - 294 \text{ dB} \]  

(1.28)

Bandwidth is an important parameter for a transducer design. Bandwidth can be calculated from the source level or transmitting voltage response. It is defined as the frequency difference of two points where the TVR is 3 dB below the peak value as shown in Fig. 1.7 [12]. It is desired to be high to widen the operating frequencies. A simple example of the calculation of quality factor can be seen in Fig. 1.8.

Quality factor, \( Q_m \), is another parameter related with the bandwidth. It calculates the sharpness of resonant response and it is defined as the ratio of the center frequency to the bandwidth. It is calculated from the TVR plot and according to 3 dB bandwidth. It can be expressed as follows:

\[ Q_m = \frac{f_p}{f_2 - f_1} \]  

(1.29)
While designing a transducer, flexural frequencies should be considered and avoided. Flexural mode can cause a null response causing one half of head mass operating out of phase with the other half [1]. Flexural frequency should be much higher than operating frequency not to affect the function of the transducer. The first flexural modes for head masses with circular and square active surfaces can be calculated respectively as:

\[
f_{flex} = \frac{1.65 ct_{flex}}{d^2(1 - v^2)^{1/2}}
\]  
\[f_{flex} = \frac{1.12 ct_{flex}}{a^2(1 - v^2)^{1/2}}
\]  

where \(v\) is Poisson’s ratio, \(t_{flex}\) is thickness of the head mass, \(c\) is the sound speed in the material assigned to head mass, \(d\) is diameter for circular surfaces and \(a\) is side length for square surfaces. Choosing the materials with high sound speed and low density is important since it causes higher flexural frequency.

**Figure 1.8** Bandwidth calculation from a TVR plot.
Beam pattern or acoustic radiation pattern is another concept used in underwater acoustic transducer design. It is described as the relative sensitivity of a transducer and used to characterize its directional response [3,15]. Beam pattern depends on the frequency and the physical dimensions of the active surface [3].

Beam pattern is generally illustrated as the reduction in sensitivity relative to the maximum sensitivity (set to zero) of a transducer as a function of spatial angle as shown in Figure 1.8 [3]. The main lobe is the region where the maximum radiation occurs. Other lobes are called sidelobes which are smaller than main lobe and occur in different directions than the main axis.

Beamwidth is the total angle between two points where the sensitivity is 3 dB less than the maximum sensitivity of the transducer on two sides of the main axis. Beamwidth depends on the dimensions of the active surface and wavelength of sound. A larger diameter of an active surface compared to the wavelength leads to a narrower beamwidth and also produce sidelobes as shown in Figure 1.9. Therefore, the transducers are generally omnidirectional at low frequencies when the wavelength is large compared to the active surface [3,15].

![Schematic description of a transducer beam pattern](image-url)
1.6 Review of Related Works

Underwater transducers have been an important research area for a long time. Because transducer design is a complex process, there have been many methods introduced in the literature. The simplest modelling technique used for a transducer design is the simple lumped parameter method. In this method, the transducer is modelled as one degree of freedom spring mass system and the resonance frequency of the transducer is calculated easily by determining natural frequency of the system. The accuracy is not good in this technique but it can be used for the initial step of design process before applying more advanced techniques [12].

ECM, first derived by Mason in 1938 [17], is another modelling method which combine the electrical and mechanical parts of the transducer in one circuit. Based on this model, various ECM have been introduced in the literature. Krimholtz et al. [18] presented alternative equivalent circuits with some advantages on particular types of piezoelectric transducers. The transducer model was represented by cascaded transmission lines in their study. Electrical input admittance calculations for an acoustic load becomes easier with this model. Cubachi and Kamata [19] presented an equivalent circuit named NKC which facilitated analysis in time and frequency domains. Dong et al. [20] introduced a six-terminal equivalent circuit based on Mason’s model considering the major losses in piezoelectric materials to obtain better analysis results. An equivalent circuit was also developed by Tilmans [21] for electromechanical transducers by using electromechanical transduction principles and equilibrium equations. Je et al. [22] presented a circuit for micromachined ultrasonic transducers and calculated the mechanical lumped parameters and radiation impedance of the transducer. They also determined the effect of piezoelectric layers on lumped parameters.

Matrix method is another technique used for transducer design. Iula et al. [23] first discovered a three-dimensional model which takes account of radial and longitudinal displacements for circular shaped piezoceramics. Then, several 3-D models are presented for different types of transducers. Iula et al. [24] presented a model also for cylinder shaped piezoceramics in which the extensional and radial vibrations are
described by differential wave equations. In addition to the models created for piezoceramics only, a matrix model of Langevin Transducer was proposed by Iula et al [25, 26]. They computed admittance of piezoceramic disk and transducer and compared them to the measured results. Mančić and Radmanović [27] presented a new three-dimensional matrix model for the piezoceramic rings and then Mančić and Stančić [28] expanded it for sandwich transducers. They compared the three-dimensional model of sandwich transducers to one dimensional models and to experimental measurements. According to resonance and antiresonance frequencies calculated by the models, the matrix model was more reliable than the one-dimensional model.

The most accurate method for transducer design is the finite element method. Dubus et al. [29] and Assaad et al. [30] analyzed ultrasonic transducers with FEM by using ATILA. FEM is applied to a Langevin transducer by using ANSYS and the behavior of the transducer in underwater in a range of 30 to 140 kHz was investigated by Iula et al. [31]. They calculated the first three natural frequencies and mode shapes of Langevin transducers with different diameter to length ratios. They stated that it was unsuitable in practical applications to use a transducer with a total length greater than its own diameter. On the other hand, Tressler et al. [32] studied the effect of the material properties and dimensions on the resonance frequency of the cymbal transducer by using ANSYS. Resonance frequencies, their mode shapes and admittance of the transducer up to 200 kHz were calculated and the results were compared to experimental measurements. It was also stated that the change of the material properties and dimensions directly affected the fundamental resonance frequency of the transducer. It was also clearly observed that ANSYS finite element program is sufficient both in air or in underwater calculations. There are several other studies about finite element analysis of sonar transducers in the literature [33, 34, 35].

Several studies regarding Tonpilz transducers are available in the literature also. Miyana et al. [36] performed equivalent circuit analysis with multiple acoustic matching plates. TVR and RVS results calculated by ECM were compared to the experimental measurements and very good agreement were obtained. The studies regarding improving the performance parameters are much in number in the recent
years. Xiping and Jing [37] studied a Tonpilz transducer with a hole in its head mass to obtain a wider bandwidth. The finite element and experimental results showed that frequency bandwidth was wider when a hole was presented in head mass. It was also shown that the hole depth and hole diameter directly affected on bandwidth and mechanical quality factor and there was an optimum dimension for the best performance parameters. Kim and Roh [38] also studied on head mass of the Tonpilz transducer to widen the bandwidth. They introduced a transducer with a void head mass to decrease the mass of the head without changing the dimension of the active surface. In order to optimize the structure, they used regression analysis and genetic algorithm. They also stated that the void head mass was effective to improve the bandwidth of the transducer. On the other hand, Butler [39] widened the TVR bandwidth by studying on the structure of the transducer. They designed two different mechanical structures to develop multi-resonant transducers. The flexural resonance is generally known as dangerous in the design of the transducers causing null response but Hawkins and Gough [40] used it to widen the bandwidth by lip-mounting.

There are studies about Tonpilz transducers to optimize the parameters like the study of Kai and De-shi [41]. They used a genetic algorithm to optimize the structure of the transducer like in the study [38]. They calculated better sound radiation power after optimization process. Saijou and Okuyama [42] optimized the phase difference of the Tonpilz transducers with a bending disk to widen the bandwidth. Non-linear goal programming algorithm was implemented by Combrugge and Thompson Jr. [43]. They introduced a systematic procedure instead of trial-and-error methods based on equivalent circuit analysis. They stated that this procedure facilitated to design wide band and high acoustic power transducers. Another study about optimization was carried out by Çiçek [44]. She optimized the design parameters of the transducer by using ECM and FEM. Optimization with ECM were not sufficient but with FEM revealed more accurate results. Teng et al. [45] made comparison of the ECM and FEM. Smith [46] discussed the ECM and boundary element methods. The design methods for transducers were clearly defined and compared to each other by Çepni [12] and Çiçek [44]. Çepni also built a design methodology starting from simple lumped parameter method and end with the experimental measurement to avoid wasting time while designing transducers.
CHAPTER 2

DESIGN METHODS AND MEASUREMENT PROCEDURES OF TRANSDUCERS

Transducers are modelled by three design methods which are simple lumped parameter method, ECM and FEM are used in this thesis. These methods and also the experimental measurement procedures are introduced in this chapter.

2.1 Simple Lumped Parameter Method

The simplest way to represent the sonar transducers is single degree of simple lumped parameter method. This model is used generally as a starting point of the design processes to have an idea before using the advanced modelling techniques. The transducer is represented as a mass-spring-damper system as shown in Figure 2.1. It is assumed that mass is ideal rigid mass and spring has only stiffness. The other parts of transducer are neglected in this model. Increasing the degrees of freedom of the model can increase the reliability of the results but it is limited because simple lumped parameter method fundamentally ignores some specific details and makes approximations [1,12].

![Mass spring system representation of a transducer](image)

**Figure 2.1** Mass spring system representation of a transducer [12]

In Figure 2.1, the terms $M_h$ and $M_t$ are analogous to head mass and tail mass respectively, the term $K_e$ is analogous to the stiffness of piezoceramic stack, and the force, $F_e$, to driving voltage. The terms $x_h$ and $x_t$ are the displacements for the head and tail masses, respectively. $R_r$ represents the radiation resistance which is a real part.
of the radiation impedance, the complex ratio of the force exerted on an active surface of a transducer caused by the vibration of the head mass to the velocity of the radiator. This is used to represent the interaction between head mass and water [12]. Radiation resistance can be defined as a measure of power transmission to the acoustic medium. The equation of motion for this system can be expressed as follows:

\[ M_h \frac{d^2 x_h}{dt^2} + (x_h - x_t)K_e + R_r \frac{dx_h}{dt} = F_e \]  

(2.1)

\[ M_t \frac{d^2 x_t}{dt^2} + (x_t - x_h)K_e = -F_e \]  

(2.2)

This system can be represented by electrical circuit as shown in Figure 2.2.

**Figure 2.2** Electrical equivalent circuit of the system shown in Figure 2.1 [1]

By assuming, \( R_r \ll \omega(M_h + M_t) \), this circuit can be converted to a single degree of freedom electrical circuit which is equivalent to SDF mass-spring-damper system as shown in Figure 2.3 [1].

**Figure 2.3** SDF mass-spring system [12]
For a single degree of freedom system shown in the Figure 2.3, the equation of motion can be written as:

\[
M_e \frac{d^2 x}{dt^2} + R_e \frac{dx}{dt} + K_e x = F
\]  

(2.3)

\(K_e\) is effective stiffness and equals to the stiffness of the piezoceramic stack in the model. The effective mass and resistance, \(M_e\) and \(R_e\) can be expressed as follows [1]:

\[
M_e = \frac{M_h M_t}{M_h + M_t}
\]  

(2.4)

\[
R_e \approx \frac{R_r}{\left(1 + \frac{M_h}{M_t}\right)^2}
\]  

(2.5)

From the equation of motion, the natural and angular natural frequencies are found as follows:

\[
\omega_n = \sqrt{\frac{K_e}{M_e}}
\]  

(2.6)

\[
f_n = \frac{1}{2\pi} \sqrt{\frac{K_e}{M_e}}
\]  

(2.7)

The response of the system for the excitation frequency, \(\omega\);

\[
x = \left(\frac{F_e}{K_e}\right) \sin \omega t
\]  

(2.8)

\[
x_{\omega = \omega_n} = \left(\frac{F_e}{R_e \omega_n}\right) = \left(\frac{F_e \omega_n}{R_e \omega_n^2}\right) = \frac{F_e \omega_n M_e}{R_e K_e} = \left(\frac{F_e}{K_e}\right) Q_m
\]  

(2.9)

\[
Q_m = \frac{\omega_n M_e}{R_e}
\]  

(2.10)

where \(Q_m\) is mechanical quality factor. Tail to head mass ratio is also used to obtain the unknowns at first;
\[ \kappa_{th} = \frac{M_t}{M_h} \]  

(2.11)

This ratio is selected typically between 2 and 4 as mentioned before although it is free to choose any number. It is preferable to select bigger values for this ratio which provides higher vibration velocity and higher acoustic power. On the other hand, to select big values may lead heavy transducer which is undesirable [1]. After tail to head mass ratio is specified, the unknowns, head mass, tail mass and stiffness of the piezoceramic stack, can be calculated from the Equations 2.4, 2.7 and 2.11.

The flexural resonance is also an important parameter to mind while calculating the dimensions of the head mass. Flexural resonance should be selected significantly higher than the resonance frequency of the transducer. Circular surfaces are generally preferred as active surface due to the advantages in terms of flexural resonance while square and rectangular surfaces are also preferable. The thickness of the head mass can be used to adjust the flexural resonance frequency. It can be found as follows, after the dimensions of the active surface is determined:

\[ t_h = \frac{M_h}{A_h \rho_h} \]  

(2.12)

where \( A_h \) is the area of the active surface and \( \rho_h \) is the density of the head mass. After the dimensions of the head mass is obtained, the material and the dimensions of the piezoceramics should be determined.

The ratio of the area of the active surface to the area of the piezoceramics is another parameter which can be used as practical value while determining the area of the ceramics. From the literature, it can be taken as [1]:

\[ \frac{A_h}{A_{pc}} = 5 \]  

(2.13)

After the area of the stack is determined, the stiffness and length of the stack can be found by estimating a value for one of them.
Lastly, the dimensions of the tail mass can be determined as follows:

\[ l_t = \frac{M_t}{A_t \rho_t} \quad (2.15) \]

### 2.2 Equivalent Circuit Method

Mechanical systems can be represented by electrical circuits by using their electrically analogous parameters. By using this method, transducers can be represented completely by electrical circuits. Transducers have electrical and mechanical parts which are working simultaneously and equivalent circuits provide visual representation of the system. In this method, all elements of the mechanical system are represented by their equivalence in the electrical system. Electrical analogous of some of the mechanical parameters are shown in Table 2.1.

<table>
<thead>
<tr>
<th>Mechanical Terms</th>
<th>Electrical Analogous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force</td>
<td>Voltage</td>
</tr>
<tr>
<td>Velocity</td>
<td>Current</td>
</tr>
<tr>
<td>Mass</td>
<td>Inductance</td>
</tr>
<tr>
<td>Damper</td>
<td>Resistance</td>
</tr>
<tr>
<td>Compliance</td>
<td>Capacitance</td>
</tr>
</tbody>
</table>

Velocity in the mechanical system is represented as current in the electrical circuit. The masses are represented as inductance, damper as resistance and reciprocal of the stiffness as capacitance. Force is equivalent to voltage. The degree of freedom of the system determines the number of circuits.

A transducer can be simply modeled as shown in Figure 2.4. In this model, as different from simple lumped parameter method, the stud and glue between ceramics can be added to the problem. The mass of the ceramics is also considered in this system.
instead of modeling them like a massless spring. The model shown in Figure 2.4 can be represented by equivalent circuit as shown in Figure 2.5.

![Figure 2.4 Simple model of a Tonpilz transducer [1]](image1)

![Figure 2.5 Electrical equivalent circuit [1]](image2)

The left side of the transducer represents the electrical part and the right side of the transducer represents the mechanical part of the transducer. The terms $R_r$ and $M_r$ indicate the acoustic part of the problem. The mechanical part has two degrees of freedom because of tail and head masses. The term $-M_{cs}/6$ stands for dynamic mass due to the vibration of the ceramics [47].

In the circuit, $R_0$ is the electrical resistance of the piezoceramics, $C_0$ is the clamped capacitance, $K_{sr}$ is the stiffness of the stress rod and $R_m$ is the resistance due to mechanical loss. The velocities are represented by voltages and $u_t$ and $u_h$ stand for velocities of the tail mass and head mass, respectively. The relative velocity can be represented by $u_r = u_h - u_t$. These velocities are related to tail and head mass as follows [1]:
Increasing the velocity of the head mass increases the radiating power and it leads to higher TVR results. It is reasonable to select large tail to head mass ratio because it leads to a higher velocity. The materials with low density can be selected for head mass and the materials with high density can be selected for tail mass to ensure this condition.

The term \( N \) is transduction coefficient or transformation ratio that relates electrical part to the mechanical part with the equation \( F = NV \). For piezoceramic transduction, the following equation including piezoelectric coefficients can be used [1]:

\[
N = \frac{F}{V} = \frac{K_{pc} \Delta t_{pc}}{E t_{pc}} = \frac{d_{33} A_{pc}}{s_{33} t_{pc}}
\]

(2.19)

Here, \( E \) is applied electric field to the piezoceramics in the direction of polarization and \( \Delta t_{pc} \) is change in thickness. \( K_{pc} \) is short circuit stiffness when \( E = 0 \).

The terms \( M_{cs}, M_{t}, M_{h} \) stand for masses of piezoceramics, tail and head, respectively. Head and tail masses are connected to each other serially and each half of the mass of the piezoceramics stack is serially connected to tail and head masses. The acoustic part of the system is also serially connected to head mass because the connection with the water is provided by the active surface of the head mass.

The terms \( K_{cs}, K_{s}, K_{g} \) stand for stiffnesses of the piezoceramics, stress rod and glue, respectively. For isotropic materials, stiffness can be found by the formula:

\[
K = \frac{A_Y}{l}
\]

(2.20)
where $A$ is the cross-sectional area, $Y$ is the modulus of elasticity and $l$ is the length of the element in stress direction. The formula can be written for piezoelectric materials which are anisotropic as:

$$K_{cs} = \frac{A_{pc}}{n_{pc} S_{33}^E t_{pc}} \quad (2.21)$$

where, $n_{pc}$ is the number of piezoceramics in the stack.

The term $C_0$ is the clamped capacitance of the transducer. It is defined as the capacitance when the motion of piezoceramics is restricted. The term $C_f$ is the capacitance of the transducer under free conditions. They can be represented mathematically as follows [1]:

$$C_f = \frac{n_{pc} \varepsilon_{33}^T A_{pc}}{t_{pc}} \quad (2.22)$$

$$C_0 = C_f (1 - k_{33}^2) \quad (2.23)$$

$$k_{33} = \frac{d_{33}}{\sqrt{S_{33}^E \varepsilon_{33}^E}} \quad (2.24)$$

Here, $k_{33}$ is the electromechanical coupling coefficient which is a measure of electromechanical performance of the transducer that converts mechanical energy to electrical or vice versa. It can be described as the ratio of converted energy to input energy [1].

In the acoustic part, $R_r$ and $M_r$ are radiation terms. The term $R_r$ is the radiation resistance which is real part of the radiation impedance and $M_r$ is the radiation mass which is imaginary part of the radiation impedance. The calculation of the radiation impedance terms for a square active surface and further information can be found in Appendix A.

$R_m$ is a mechanical resistance of the transducer and desired to be low compared to the radiation resistance because of its effect on the performance of the transducer. Mechanoacoustic efficiency is related to $R_m$ and $R_r$ by the following equation:
$$\eta_{ma} = \frac{R_r}{R_r + R_m} \quad (2.25)$$

$R_0$ is the electrical loss resistance which is electrical resistance of the piezoceramics modelled as capacitors in the equivalent circuit. It is defined as:

$$R_0 = (\omega C_f \tan \delta)^{-1} \quad (2.26)$$

Here, $\tan \delta$ is the electrical dissipation factor and for piezo ceramics typically between 0.004 and 0.02.

This two-degrees of freedom circuit can be solved as it is or can be reduced to one degree of freedom to simplify the problem with the assumption of $\omega(M_h + M_t + M_s + M_r) \gg R_r + R_m$, often the case in practice. The effective masses, resistances and compliances are derived for the resulting one-degree freedom of the system as [1]:

$$M_e = \frac{M_1}{1 + \frac{M_1}{M_2}} - \frac{M_{cs}}{6} \quad (2.27)$$

$$R_e = \frac{R_r + R_m}{\left(1 + \frac{M_1}{M_2}\right)^2} \quad (2.28)$$

$$C_e = \frac{C_{cs}}{1 + \frac{C_{cs}}{C_{sr}}} \quad (2.29)$$

where the terms $M_1$ and $M_2$ are:

$$M_1 = M_r + M_h + \frac{M_{cs}}{2} \quad (2.30)$$

$$M_2 = M_t + \frac{M_{cs}}{2} \quad (2.31)$$

Then the mechanical impedance and the total impedance of the circuit can be written as follows:
\[ Z_m = \left( \frac{1}{j\omega C_e} + j\omega M_e + R_e \right) \]  
(2.32)

\[ Z = R_0 + j\omega C_0 + N^2 / Z_m \]  
(2.33)

For a sinusoidal input of \( V \), the relative velocity is calculated by:

\[ u_r = \frac{NV}{Z_m} \]  
(2.34)

After finding relative velocity, the velocity of the head mass can be calculated from relations of the mass velocities. Then the far field pressure in the acoustic axis can be calculated from Eq. 1.16.

### 2.3 Finite Element Method

Finite Element Method (FEM) is a numerical technique used to solve complex engineering and scientific problems. It is used to find an approximate solution to a problem where an exact solution is difficult [48]. FEM is based on discretization of the problem to a finite number of elements. Combination of the simple equations of these small elements forms the solution for the entire problem [49].

FEM has been applied many engineering areas like structural mechanics, fluid dynamics, heat transfer, electromagnetism and in mathematical applications like boundary value problems and numerical solutions of ordinary and partial differential equations [48]. Actually, it is generally very difficult to find an analytical solution to an engineering unless the problem is not significantly simplified [12]. For example, the head masses with square shaped active surfaces which is used in this study do not have an analytical solution although it is used for a long time in transducer applications [12]. Thus, it can be said that the application of the FEM is a must in the engineering problems.

FEM is based on discretization of the problem, meshing in other words. So, it only offers an approximate solution rather than an exact solution. The accuracy of the solution mostly depends on the meshing quality if the problem is truly modeled.
Increasing number of elements by lowering mesh size can improve solution quality but it will increase the equations to be solved and accordingly the time for the solution.

FEM generally consists of the following steps. First, the problem is divided to a finite number of elements. A solution or interpolation model is selected and then stiffness and load matrices are derived. The overall equilibrium equation is then obtained with the combination of the element equations. These equations are solved for the unknowns and then element stresses and strains are found.

Piezoelectric elements are difficult to model in FEM because they are anisotropic materials. The equations 2.35 and 2.36 which define the stress and electrical displacement vectors of piezoelectric materials can be used to include piezoelectric materials in the finite element calculations [1].

\[ \sigma = c^E \varepsilon - e^t E \]  \hspace{1cm} (2.35)

\[ D = e \varepsilon + \epsilon^e E \]  \hspace{1cm} (2.36)

These equations can be written as:

\[ F_t = K^E x - NV \]  \hspace{1cm} (2.37)

\[ Q = N^t x + C^S V \]  \hspace{1cm} (2.38)

where \( K^E \) is short-circuit stiffness matrix, \( Q \) is the charge vector, \( C^S \) is the clamped electrical capacitance matrix, \( V \) is voltage, \( F_t \) is the total force vector and can be written as follows [1].

\[ F_t = F_b - M \frac{d^2 x}{dt^2} - R \frac{dx}{dt} \]  \hspace{1cm} (2.39)

Eq. 2.37 and Eq. 2.38 can be rewritten as follows [1].

\[ M \frac{d^2 x}{dt^2} + R \frac{dx}{dt} + K^E x - NV = F_b \]  \hspace{1cm} (2.40)
\[ \frac{d^2x}{dt^2} + \frac{dx}{dt} + N^T x + C^S V = Q \]  

(2.41)

The above equations can be represented by the matrix equation as follows [1].

\[
\begin{bmatrix}
M & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
\dot{x} \\
\dot{\psi}
\end{bmatrix} + \begin{bmatrix}
R & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
x \\
\psi
\end{bmatrix} + \begin{bmatrix}
K^E & -N \\
N^T & C^S
\end{bmatrix} \begin{bmatrix}
x \\
V
\end{bmatrix} = \begin{bmatrix}
F_b \\
Q
\end{bmatrix}
\]  

(2.42)

In Eq. 2.42, all matrices have sub-matrices in each. This equation couples the mechanical and piezoelectric elements by electromechanical coupling ratio \( N \). This is a simple problem that can be solved by FEM. The real problems are more than one dimension and required to have very much elements to obtain accurate solutions. Because of this reason, computer programs are developed based on the FEM. For the solution of the transducer applications, Multiphysics programs are required. Transducers have structural and acoustic parts and they should be solved simultaneously. Some of the finite element analysis (FEA) programs are capable of solving piezoelectric problems at underwater conditions. COMSOL Multiphysics is very useful for the transducer and piezoelectric applications and capable of solving Multiphysics problems. FEA in COMSOL is generally performed by the following steps [50];

1. First, selection of the space dimensions, physics interfaces and study type in the model wizard section should be done. For transducer analysis, pressure acoustics, electrostatics and solid mechanics can be used as physics interfaces with frequency domain study type in 3-D or 2-D. The acoustic-piezoelectric interaction, Frequency Domain interface which includes the necessary interfaces mentioned before can also be used rather than selecting them one by one.

2. The geometry should be drawn or be imported after model wizard.

3. The domains for each physics interfaces should be selected and the settings of the domains should be specified according to the requirements.

4. Assigning the materials to the domains is the next step for the analysis. The materials can be selected from the library as well as can be user defined.
5. After the material properties are assigned, the model should then be meshed. The element size and type can be set by the user or the standard mesh can be used. The mesh quality affects the computation time and accuracy in the problem, so it should be selected properly.

6. The loads to be applied should be selected.

7. In the study type section, the solution concept should be decided. For transducer analysis in the frequency domain, the range of frequencies in which the analysis will be performed is a parameter to be decided.

8. In the last step, after the solution is done, the results can be studied in the results section.

An example of a Tonpilz type transducer modelled in COMSOL is given in Figure 2.6 [51]. This model is referenced from a tutorial of COMSOL and the modelling and solution steps can be investigated in detail from the tutorial paper [51]. The Acoustic-Piezoelectric Interaction, Frequency Domain interface was used for this analysis. It was a 3-D analysis and an axisymmetric view of the actual 3-D transducer is shown in Figure 2.6. The transducer’s main parts were modeled and water domain was defined in front of the active surface. Here, a PML (Perfectly Matched Layer) domain was modeled in the system. PML was used to provide the infinity of the water domain.

![Figure 2.6](image_url) Quarter section of Tonpilz transducer in COMSOL [51]
The waves generated by the transducer enter water domain and then they disappear in the infinity. However, because the finite element model built for analysis is not infinite, the waves are reflected from the outer surface of the domain. In order to avoid this condition, an absorber surrounding the area should be defined for the generated waves. The perfectly matched layer solves this problem using the coordinate transformation to act as a material which dampens the wave not to cause any reflections [51].

After transducer was modelled, the materials were assigned to the domains and the model was meshed. 1 V rms electrical signal was applied for each piezoceramics from the positive direction and the other sides of the piezoceramics were grounded as seen in Figure 2.7. For a given frequency range, the problem was solved in the frequency domain. In the postprocessing, the required results such as TVR or pressure distribution of the water were viewed.

![Figure 2.7 Applied voltage and polarization directions [51]](image)

2.4 Calibration and Measurement of the Transducers

Transducers should be calibrated in advance. Calibration is a process to specify the characteristic properties of the transducer such as TVR, RVS, impedance and beam pattern with respect to the frequency range.

There are two different methods, primary and secondary methods, for the calibration of the transducers. Primary methods require only measurement of the electrical and
acoustic impedance, voltage, current, length, density and frequency for the calibration. The primary methods do not need a calibrated transducer while the secondary methods use a reference transducer calibrated with primary methods. The secondary methods are preferred mostly for routine calibrations because they require less measurements than the primary methods [52].

Generally, high quality transducers with broad bandwidth are calibrated using the primary methods and they are used as reference transducers in the secondary methods [52].

2.4.1 Primary Methods

The most useful primary method is conventional reciprocity method (three transducers spherical-wave reciprocity). In this method, three transducers are matched in three different measurement instrument. One of the transducers is a projector, one is hydrophone and one is a reciprocal transducer. For reciprocal transducers, the ratio of receiving sensitivity, $M_r$, to transmitting voltage response, $S$, is constant. This constant is known as reciprocity parameter and can be formulated as follows [52]:

$$f = \frac{M_r}{S} = \frac{2r_0}{\rho f}$$

(2.43)

where $r_0$ is a reference far field distance usually 1 m, $\rho$ is density of medium and $f$ is frequency. Here, it should be known that the transmitting voltage response is the ratio of the acoustic pressure in a distance $r$ to the input current. Hence, the pressure in a distance $r$ from a projector driving with current $i$ can be formulated as [52].

$$P = Si \frac{r_0}{r}$$

(2.44)

The transfer impedance for a projector-hydrophone couple is the open-circuit voltage of hydrophone to the input current of projector and can be formulated as follows.

$$Z = \frac{V_H}{i_p}$$

(2.45)
Procedure:

Three transducers are located as seen in Figure 2.8. The notations P, H and R represent a projector, hydrophone and reciprocal transducer, respectively. In first measurement, a hydrophone is located at a distance $r_1$ from the projector. The open-circuit voltage of the hydrophone when the projector is driven with an input current $i_{PH}$, can be formulated as follows [52]:

$$V_{PH} = \frac{M_H S_P i_{PH}}{r_1}$$  \hspace{1cm} (2.46)

then, the transfer impedance is found as [52].

$$Z_{PH} = \frac{M_H S_P}{d_1}$$  \hspace{1cm} (2.47)

For the second measurement, a reciprocal transducer is used instead of a hydrophone at a distance $r_2$ from the projector and in the third measurement, a hydrophone is located at a distance $r_3$ from the reciprocal transducer. As similar to the first measurement the transfer impedances are found as [52]:

$$Z_{PR} = \frac{M_R S_P}{d_2}$$  \hspace{1cm} (2.48)
There are four equations including reciprocity equation (2.43, 2.47, 2.48 and 2.49) and four unknowns \( \mathcal{M}_R, \mathcal{M}_H, S_R, S_P \) to solve. The system of equations can be solved easily for the unknowns and then acoustic parameters can be calculated. For example, the free field sensitivity of the hydrophone is found as follows:

\[
\mathcal{M}_H = \sqrt{\frac{d_1 d_2 Z_{PH} Z_{RH}}{d_3 Z_{PR}}} \tag{2.50}
\]

The fourth measurement can be performed if the projector and hydrophone are reciprocal as well. This measurement is known as reciprocity control and used to check the accuracy of the method. In Eq. 2.50, instead of \( Z_{PR} \), \( Z_{RP} \) can be written and the accuracy of the result can be checked in this way.

### 2.4.2 Secondary Methods

For the hydrophone measurement, a previously calibrated hydrophone and the hydrophone to be calibrated are subjected to the same free field pressure and the open-circuit voltages of the hydrophones are measured. If the calibrated hydrophone is not omni-directional, acoustic axis of the hydrophone should be directed to the projector [52].

The open circuit voltage of the standard hydrophone \( V_s \) and the open circuit voltage of the unknown hydrophone \( V_x \) are measured respectively at the same position. The free field sensitivity of the unknown hydrophone, \( \mathcal{M}_x \), can be found by knowing the free field sensitivity of a standard hydrophone, \( \mathcal{M}_s \), as follows [52]:

\[
\mathcal{M}_x = \frac{\mathcal{M}_s V_x}{V_s} \tag{2.51}
\]

For a projector calibration, a calibrated reference hydrophone with free field voltage sensitivity \( \mathcal{M}_H \) is located to a distance \( r \) from the projector. A driving voltage \( V_P \) is
applied to the projector and open circuit voltage $V_H$ of the hydrophone is measured. Then, transmitting voltage response of the projector is found as follows [52]:

$$S_p = \frac{V_H r}{V_P M_H} = \frac{Z_{PH} r}{Z_P M_H}$$

(2.52)

2.4.3 Measurement Facilities

Transducer measurements are performed in natural sites like lakes, ponds, ocean inlets and also in artificial pools and indoor pressure tanks. The requirements of the facilities for measurement of transducers are basically enough space to eliminate boundary reflections, low ambient noise and a water medium free of marine life, bubbles or pollutants to eliminate sound refraction and scattering [52].

Far-field distance is a parameter to calculate space requirement of the measurement facilities. The parameters of the transducers are calculated in the far-field distance as mentioned in Chapter 1. All measurement techniques should provide this condition to obtain accurate results. Far-field distance depends on the active surface and the wavelength of the emitted wave by the following formula:

$$r_{ff} \geq \frac{A}{\lambda}$$

(2.53)

where, $r_{ff}$ is far field distance, $A$ is the area of active surface and $\lambda$ is the wavelength.

For all measurement techniques, transducers are submerged to a water medium such as pools and lakes. A sound wave is created underwater by a projector and this wave is received by a hydrophone. While this sound wave is spreading, it reflects from the boundaries of the site and they also are received by hydrophone. This disturbs the sound wave emitted from the projector and cause errors in the measurements. Therefore, the site dimensions are important in measuring the transducer characteristics. Figure 2.9 shows a schematic diagram of the echoes reflected from boundaries in a water tank.
Figure 2.9 Main sources of reflections in a measurement in water tank [53]

Figure 2.10 shows a measurement data of a hydrophone for two different frequencies. The first signal become steady after the transient region in the beginning three or four cycles. In the steady state region, the signal is contaminated at the moment of the first echo is reached to the hydrophone. After that, it is not possible to make accurate measurements. The time echo reached to the hydrophone depends on the dimensions of the measurement site. Therefore, the dimensions of the measurement site should be large enough to provide enough time for the analysis of the signal [53].

Figure 2.10 Measured data for a transducer (ITC1001) corresponding to 2 kHz (left) and 20 kHz (right) drive voltage [53]
The frequency of the analysis is a parameter to determine the dimensions of the site. The period of once cycle of the signals with lower frequencies take longer time than the higher frequencies. In other words, less cycle can be analyzed before the signal is contaminated as seen in Figure 2.10. Therefore, larger dimensions of measurement site are required for the analysis of lower frequencies. In the data shown in Figure 2.10, there is enough time to analyze the signal before contaminated. Typically, a gate or time window is applied to take only the uncontaminated steady state signal data for the analysis. In this way, the steady state part of the signal is gated and the part after the echo is reached to the hydrophone is not analyzed [52, 53].

After gated data is obtained, the amplitude of the signal is measured in several ways as follows [53];

i- Measuring directly the peak voltage of the signal with peak detector.
ii- Calculating the root-mean-square voltage.
iii- Performing fast Fourier transform (FFT) of the signal and obtaining amplitude of the spectrum at the drive frequency.
iv- Performing ‘narrow-band’ discrete Fourier transform (DFT) of the signal and obtaining amplitude of only the component at the drive frequency.
v- Performing least-squares method to fit the sine wave of the appropriate frequency and taking the amplitude of the fitted sine-wave.

All the methods stated above have advantages and disadvantages. The peak measurement is simple but inaccurate in the presence of any noise and distortion while the others require more processing and software algorithms. The methods from ii to iv should be performed with an integer number of cycles to have accurate results. The methods iv and v are useful when only a half cycle of signal is analyzed [53].

In addition to the physical and environmental requirements of the facilities, some basic materials are needed to perform the analysis of the signal. First, the transducers should be located in true position. In order to obtain a sound wave, the projector should be driven by a signal. This signal is amplified by power amplifier and it is transferred to the piezoceramics. Then, it is transformed to the acoustic energy by the vibration of piezoceramics and a sound wave is emitted to the water. After that, the hydrophone
takes this acoustic energy and transform it to electrical signal by the help of piezoceramics. This signal is amplified and gated. This gated signal is transferred to the oscilloscope and it is analyzed with a computer connected to the oscilloscope by one of the analysis methods mentioned before. Besides, an impedance analyzer is required for the impedance calculations of the transducers.

Shortly, minimum requirements of a facility performing measurement of transducers can be said as follows;

- Enough space to provide far-field distance and to eliminate boundary reflections
- Low ambient noise and a water medium free of anything causing reflection
- Positioning system
- Signal generator
- Power amplifier
- Pre-amplifier
- Oscilloscope
- Computer
- Impedance analyzer
CHAPTER 3

DESIGN OF THE TRANSDUCER

A Tonpilz transducer working below 7 kHz is designed by following the design steps introduced in Chapter 2. The design methodology starts with defining of the design criteria and initial parameters, and ends with experimental analysis. Firstly, the design criteria are specified and according to these criteria, geometrical shape and materials are defined. Initial design steps start with the simple lumped parameter method and continue with ECM and FEM, respectively. Rough dimensions are obtained with the simple lumped parameter method. After that, optimization on head and tail masses performed in ECM. After optimum values of head and tail masses are found, another optimization based on dimensions and numbers of piezoceramics are performed with finite element method. At the end of the design procedure, three transducers which perform better acoustic performance are selected for the manufacturing step.

3.1 Design Parameters

The first step of the design procedure is to define the design criteria of desired transducer. In this study, the most important characteristics of the transducer are operating frequency, bandwidth and TVR level. Transducer is desired to operate near or below 7 kHz with a maximum quality factor of 5. Also, minimum TVR in operating frequency is required to be 130 dB. In this thesis, quality factor is calculated from 3-dB bandwidth of TVR. The peak frequency of TVR corresponds to the operating frequency of the transducer.

RVS and beamwidth are also the other important parameters and they should be viewed in the design processes. Maximum RVS over -160 dB and beamwidth about 90 degrees are sufficient.

Transducer type is a tonpilz-type which is used generally to obtain relatively low frequency and high acoustic power. The transducer is desired to be as small as possible. However, reducing the dimensions generally increases the natural frequency of the
transducer. Therefore, optimization is the crucial step in the design process to find the optimum working frequency within the limited dimensions.

Transducer has some physical dimensional limitations also. Active surface of the head mass should be in limits of a 100x100 mm square including rubber coating. So, the maximum dimension for the active surface is about 96x96 mm. The length of the transducer is also limited and should be about 120 mm. The dimensions are important and making smaller the transducer is preferred.

After the design parameters are specified, materials for the transducer parts are defined. The materials widely used in the literature are applied for the parts. Aluminum is used for head mass, steel is used for tail mass and stress bolt, and PZT-4 for piezoceramics. According to these parameters, design steps are started with the simple lumped parameter method.

The values specified in this chapter are prerequisites of this study. They are chosen according to requirements of Defense Industry applications. Also, they are compatible with used platforms in underwater applications.

### 3.2 Simple Lumped Parameter Method

Simple lumped parameter method is used as the first step for the initial design of the transducer. The results may not be much reliable but it is the basic step to find rough dimensions and to prepare some assumptions for the ECM and FEM.

First step for the simple lumped parameter method is to specify the dimensions of the active surface (e.g., head mass). To ensure the limits for the head mass, a square with a 96 mm edge is selected for the active surface. The minimum width of the head mass should be determined from the flexural modes of the transducer. Because the first flexural mode should be much higher than the natural frequency of the transducer, it is assumed to be 15 kHz which is higher than two times of desired natural frequency. Since aluminum is assigned to the head mass, the speed of sound in aluminum is taken 6320 m/s. From Eq. 1.31:
\[ 15000 \, Hz = \frac{1.12 \left( \frac{6320 \, m}{s} \right) t_{flex}}{(0.096 \, m)^2 (1 - 0.33^2)^2} \]

\[ t_{flex} \geq 18 \, mm \]

The thickness of the head mass is selected as 18 mm. Knowing the density of the aluminium, the mass of the head can be obtained from Eq. 2.12 as follows:

\[ M_h = A_h t_h \rho_h = 0.46 \, kg \]

By selecting tail to head mass ratio \( \kappa_{th} = 4 \), it is found that \( M_t = 1.84 \, kg \). From Eq. 2.4 and Eq. 2.7, equivalent stiffness can be found as follows:

\[ M_e = \frac{M_h M_t}{M_h + M_t} = 0.37 \, kg \]

\[ 7000 \, Hz = \frac{1}{2\pi} \sqrt{\frac{K_e}{0.37 \, kg}} \]

\[ K_e = 0.7 \times 10^9 \, N/m \]

By assuming \( l_{pc} = 60 \, mm \) initially, area of the piezoceramics can be found from Eq. 2.14 as follows:

\[ 0.7 \times 10^9 \, N/m = \frac{A_{pc}}{(0.06 \, m)(15.5 \times 10^{-12} \, m^2/N)} \]

\[ A_{pc} = 6.5 \times 10^{-4} \, m^2 \]

Therefore, the inner and outer radii of the piezoceramics can be assumed to be 21 mm and 25.4 mm, respectively.

If the length of the tail mass is chosen as \( l_t = 40 \, m \) to ensure the requirements about the length of the transducer, area of the tail can be determined from Eq. 2.15 as follows:

\[ A_t = \frac{M_t}{l_t \rho_t} = \frac{1.84 \, kg}{(0.04 \, m)(7845 \, kg/m^3)} = 5 \times 10^{-3} \, m^2 \]
Figure 3.1 shows the rough dimensions of the transducer according to calculated values in simple lumped parameter method.

Here, rough dimensions of the stress rod are selected as $d_{sr} = 12\, mm$ and $l_{sr} = 110\, mm$, considering the physical limitations and the dimensions of the other parts. The dimensions obtained from the first step of the design process are summarized in Table 3.1.

**Table 3.1** Dimensions of parts obtained in first step

<table>
<thead>
<tr>
<th>Dimension [mm]</th>
<th>Step 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>96</td>
</tr>
<tr>
<td>$t_h$</td>
<td>18</td>
</tr>
<tr>
<td>$r_{pc,o}$</td>
<td>25.4</td>
</tr>
<tr>
<td>$r_{pc,i}$</td>
<td>21</td>
</tr>
<tr>
<td>$l_{cs}$</td>
<td>60</td>
</tr>
<tr>
<td>$r_{t,o}$</td>
<td>42</td>
</tr>
<tr>
<td>$l_t$</td>
<td>40</td>
</tr>
<tr>
<td>$l_{sr}$</td>
<td>110</td>
</tr>
<tr>
<td>$r_{sr}$</td>
<td>12</td>
</tr>
</tbody>
</table>
3.3 Equivalent Circuit Method

After the rough dimensions are initially obtained, the transducer is analyzed with more accurate design method, ECM. Parameters obtained from the first step are used directly in this step and then with parameter sweep, the analysis is done for several times to find an optimum solution.

The tail mass, head mass and stiffness of the piezoceramics are known from simple lumped parameter method. Clamped capacitance, stiffness of the stress rod, radiation mass and radiation resistance should be found before starting the analysis.

In order to find the clamped capacitance, the number of the piezoceramics should be known. The number of ceramics is initially selected to be 6, resulting in 10 mm thickness for each ceramic. From Eqs. 2.22, 2.23 and 2.24:

\[ C_0 = \frac{n_{pc}e_{33}^2A_{pc}}{t_{pc}}(1 - k_{33}^2) = \frac{6 \left( 11.5 \times 10^{-9} \frac{C}{mV} \right) \left( 6.41 \times 10^{-4} m^2 \right)}{10 \times 10^{-3} m} (1 - 0.68^2) \]

\[ C_0 = 2.38 \text{nF} \]

The stiffness of the stress rod can be found from the Eq. 2.20, according to the dimensions obtained in previous step as follows:

\[ K_{sr} = \frac{A_{sr}Y_{sr}}{l_{sr}} = \frac{(1.13 \times 10^{-4} m^2) \left( 215 \times 10^9 N/m^2 \right)}{0.11 m} = 0.22 \times 10^9 N/m \]

The total mass of piezoceramic stack is found as follows:

\[ M_{cs} = A_{pc}l_{cs}\rho_{pc} = (6.41 \times 10^{-4} m^2) (60 \times 10^{-3} m) \left( 7500 \frac{kg}{m^3} \right) = 0.29 \text{ kg} \]

The transformation ration can be found from Eq. 2.19:

\[ N = \frac{d_{33}A_{pc}}{s_{33}^e \epsilon_{pc}} = \frac{\left( 289 \times 10^{-12} \frac{C}{N} \right) \left( 6.41 \times 10^{-4} m^2 \right)}{\left( 15.5 \times 10^{-12} \frac{m^2}{N} \right) (0.01 m)} = 1.2 \]
The radiation terms, $R_r$ and $M_r$, are determined with respect to the frequency. The equivalent circuit including obtained parameters is shown in Figure 3.2. The circuit is solved by using mesh current methods. The voltage in the mechanical part of the circuit equals to driven voltage times transduction ratio. The circuit turns into a form as shown in Figure 3.3 after eliminating electrical part.

![Figure 3.2 Equivalent circuit of the transducer](image1)

![Figure 3.3 Equivalent circuit of the transducer](image2)

In order to find the current passing through the head mass, the circuit loops of the system are solved with Matlab. Here, $u_2$ refers to the velocity of the head mass. From Equations 1.5 and 1.16, the pressure magnitude at the far field and TVR is calculated in a range of frequency from 3 kHz to 13 kHz with a step size of 0.01 kHz.
The maximum TVR is determined at 6.1 kHz with a quality factor of 3.8 for the initial values. The results are good for the desired transducer but a small optimization is done to see other solutions. The analyses are performed for different values of mass parameters $M_h$ and $M_t$ which strongly influence the resonance frequency. The results are shown in Table 3.2.

Three different values ($M_t = 1.84, 1.5, 1 \, kg$) for tail mass and 6 different values for head mass are used for the analyses. Bigger or smaller values are not considered because of the physical limitations. On the other hand, smaller step sizes can be used to see more detailed results but it is not effective in this system because the parameters cannot be exactly provided in the actual design. The approximate dimensions for the transducer parts are sufficient in this step.

<table>
<thead>
<tr>
<th>$M_t(kg)$</th>
<th>$M_h(kg)$</th>
<th>$f_p(kHz)$</th>
<th>TVR(dB)</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.84</td>
<td>0.86</td>
<td>5.6</td>
<td>138.5</td>
</tr>
<tr>
<td>2</td>
<td>1.84</td>
<td>0.76</td>
<td>5.8</td>
<td>138</td>
</tr>
<tr>
<td>3</td>
<td>1.84</td>
<td>0.66</td>
<td>5.9</td>
<td>138</td>
</tr>
<tr>
<td>4</td>
<td>1.84</td>
<td>0.56</td>
<td>6.2</td>
<td>137.5</td>
</tr>
<tr>
<td>5</td>
<td>1.84</td>
<td>0.46</td>
<td>6.4</td>
<td>137</td>
</tr>
<tr>
<td>6</td>
<td>1.84</td>
<td>0.36</td>
<td>6.8</td>
<td>136.5</td>
</tr>
<tr>
<td>7</td>
<td>1.4</td>
<td>0.86</td>
<td>5.9</td>
<td>139</td>
</tr>
<tr>
<td>8</td>
<td>1.4</td>
<td>0.76</td>
<td>6.1</td>
<td>139</td>
</tr>
<tr>
<td>9</td>
<td>1.4</td>
<td>0.66</td>
<td>6.3</td>
<td>138.5</td>
</tr>
<tr>
<td>10</td>
<td>1.4</td>
<td>0.56</td>
<td>6.5</td>
<td>138</td>
</tr>
<tr>
<td>11</td>
<td>1.4</td>
<td>0.46</td>
<td>6.8</td>
<td>137.5</td>
</tr>
<tr>
<td>12</td>
<td>1.4</td>
<td>0.36</td>
<td>7.1</td>
<td>137</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>0.86</td>
<td>6.5</td>
<td>140</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>0.76</td>
<td>6.6</td>
<td>139.5</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>0.66</td>
<td>6.8</td>
<td>139.5</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>0.56</td>
<td>7</td>
<td>139</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>0.46</td>
<td>7.3</td>
<td>138.5</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>0.36</td>
<td>7.6</td>
<td>138</td>
</tr>
</tbody>
</table>
For the sake of simplicity of the analysis, some parts of the transducer like glue, electrodes and insulators are excluded in the ECM. These parts are included to the circuit as capacitances connected to the piezoceramics in parallel to improve the results. The results for this case can be seen in Table 3.3. Including parallel capacitances to the circuit means decreasing the total capacitance of the circuit. Capacitance decrease in the circuit leads to a decrease in peak frequency and bandwidth. Therefore, peak frequencies obtained by the same parameters decrease after adding passive elements to the circuit. Bandwidth of the TVR also decreases and as a result of this, quality factor increases.

Table 3.3 Parameters obtained from ECM (improved)

<table>
<thead>
<tr>
<th></th>
<th>M&lt;sub&gt;t&lt;/sub&gt;(kg)</th>
<th>M&lt;sub&gt;h&lt;/sub&gt;(kg)</th>
<th>f&lt;sub&gt;p&lt;/sub&gt;(kHz)</th>
<th>TVR(dB)</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.84</td>
<td>0.86</td>
<td>5.1</td>
<td>139</td>
<td>8.5</td>
</tr>
<tr>
<td>2</td>
<td>1.84</td>
<td>0.76</td>
<td>5.2</td>
<td>138.5</td>
<td>7.5</td>
</tr>
<tr>
<td>3</td>
<td>1.84</td>
<td>0.66</td>
<td>5.4</td>
<td>138</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>1.84</td>
<td>0.56</td>
<td>5.6</td>
<td>138</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>1.84</td>
<td>0.46</td>
<td>5.8</td>
<td>137.5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>1.84</td>
<td>0.36</td>
<td>6.1</td>
<td>137</td>
<td>3.1</td>
</tr>
<tr>
<td>7</td>
<td>1.4</td>
<td>0.86</td>
<td>5.4</td>
<td>139.5</td>
<td>8.3</td>
</tr>
<tr>
<td>8</td>
<td>1.4</td>
<td>0.76</td>
<td>5.5</td>
<td>139</td>
<td>7.9</td>
</tr>
<tr>
<td>9</td>
<td>1.4</td>
<td>0.66</td>
<td>6.3</td>
<td>139</td>
<td>6.3</td>
</tr>
<tr>
<td>10</td>
<td>1.4</td>
<td>0.56</td>
<td>5.9</td>
<td>138.5</td>
<td>5.4</td>
</tr>
<tr>
<td>11</td>
<td>1.4</td>
<td>0.46</td>
<td>6.1</td>
<td>138</td>
<td>4.4</td>
</tr>
<tr>
<td>12</td>
<td>1.4</td>
<td>0.36</td>
<td>6.4</td>
<td>137</td>
<td>3.2</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>0.86</td>
<td>5.9</td>
<td>140.5</td>
<td>9.8</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>0.76</td>
<td>6</td>
<td>140</td>
<td>8.6</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>0.66</td>
<td>6.2</td>
<td>139.5</td>
<td>6.9</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>0.56</td>
<td>6.3</td>
<td>139</td>
<td>5.7</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>0.46</td>
<td>6.6</td>
<td>138.5</td>
<td>4.4</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>0.36</td>
<td>6.9</td>
<td>138</td>
<td>3.3</td>
</tr>
</tbody>
</table>

From Table 3.3, the 5<sup>th</sup> set is selected with natural frequency of 5.8 kHz and quality factor of 4. The head and tail masses are $M_h = 0.46 \text{ kg}$ and $M_t = 1.84 \text{ kg}$ for this set.
The TVR of the transducer obtained with (ECM-improved) and without (ECM-basic) passive elements is shown in Figure 3.4.

![Figure 3.4 TVR results calculated by ECM](image)

In this step, the dimensions of the main parts are determined according to the calculated parameters. The simple view of the transducer for the values obtained in second step of the design process can be seen in Figure 3.5. In the next step, the FEM is used to compare the results and to improve the parameters by changing the dimensions and numbers of the piezoceramics.

![Figure 3.5 Cross sectional view of transducer obtained in the second step](image)
3.4 Finite Element Method

The analysis and design parts of the transducers are performed using COMSOL and SOLIDWORKS. COMSOL is used as the finite element analysis program to find the optimum design. Drawing the transducer before the analysis and dimensional parameter definitions are done in SOLIDWORKS software due to its advances about modelling and designing.

COMSOL is a Multiphysics software that can perform multiple physics simultaneously. For example, in this study, structural and acoustic analysis should be combined because structural change in the transducer causes acoustic results such as acoustic waves and acoustic pressures. Therefore, Acoustic Module which interacts acoustic and structural physics is used for the analysis. This module provides necessary interfaces needed for the transducer analysis.

First of all, transducer is constructed with the SOLIDWORKS as shown in Figure 3.6. The main transducer parts such as head, tail, piezoceramics, stress rod and nut are modeled. The glue and electrodes between ceramics are skipped in the model.

Figure 3.6 3D model of the transducer constructed with Solidworks
After a 3-D model of the transducer is obtained, the physics interfaces; pressure acoustics, electrostatics and solid mechanics, are selected in COMSOL. The study type is selected as a frequency domain because the transducer characteristics will be determined with respect to the frequency.

Then, transducer model is transferred to the COMSOL Multiphysics by the live link option to perform the analysis. Live link is an option in COMSOL which provides simultaneous connection with another software program like SOLIDWORKS and any change in the program is directly received by COMSOL. After the transducer is imported, the water and PML domains are defined in front of the active surface. The materials for each part and domain are assigned. The model in COMSOL and assigned materials are shown in Figure 3.7.

The piezoceramics should be poled before applying voltage. Piezoceramics are poled in +Z and -Z directions, respectively. +Z polarization is directly obtained from the global coordinate system. -Z polarization is obtained with the new coordinate system with opposite direction of the Z coordinate according to default coordinate system. After polarization is defined, 1 V rms is applied to the positive surfaces and the other surfaces are grounded as shown in Figure 3.8.
After the materials are assigned to the parts, the model is meshed. The water domain is meshed according to the minimum wavelength. Tetrahedral elements are used in meshing. Maximum element size used in analysis is 10 mm and minimum element size is 0.1 mm. Average mesh quality is calculated as 0.727 which is a sign of good quality of meshing. The transducer after meshing process is shown in Fig. 3.9.
For the selected parameters from the ECM, the peak value for the frequency is found at 5.8 kHz with a quality factor of 4. The analysis is run in a range of frequency from 3 kHz to 13 kHz with a step size of 0.1 kHz. The analysis is performed for 6 and 8 piezoceramics and in a range from 19 mm to 25.4 mm for inner and outer radii, respectively. TVR of the transducer for different size and number of piezoceramics are calculated and the determined peak frequency and quality factor are compared on Table 3.4.

Table 3.4 Results obtained with Finite Element Method

<table>
<thead>
<tr>
<th>n</th>
<th>( r_{pc_a} ) (mm)</th>
<th>( r_{pc_i} ) (mm)</th>
<th>( l_{pc} ) (mm)</th>
<th>( f_p ) (kHz)</th>
<th>TVR (dB)</th>
<th>( f_2 ) (kHz)</th>
<th>( f_1 ) (kHz)</th>
<th>( Q_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25.4</td>
<td>19.05</td>
<td>10.8</td>
<td>6.3</td>
<td>139.5</td>
<td>7.5</td>
<td>5.6</td>
<td>3.3</td>
</tr>
<tr>
<td>2</td>
<td>25.4</td>
<td>20</td>
<td>10.8</td>
<td>6</td>
<td>6.0</td>
<td>138.5</td>
<td>7.1</td>
<td>5.4</td>
</tr>
<tr>
<td>3</td>
<td>25.4</td>
<td>21</td>
<td>10.8</td>
<td>6</td>
<td>5.6</td>
<td>137.5</td>
<td>6.5</td>
<td>5.1</td>
</tr>
<tr>
<td>4</td>
<td>25.4</td>
<td>22</td>
<td>10.8</td>
<td>6</td>
<td>5.2</td>
<td>136</td>
<td>6</td>
<td>4.7</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>19.05</td>
<td>10.8</td>
<td>6</td>
<td>6.1</td>
<td>139</td>
<td>7.2</td>
<td>5.5</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
<td>19.05</td>
<td>10.8</td>
<td>6</td>
<td>5.5</td>
<td>137.5</td>
<td>6.5</td>
<td>5.1</td>
</tr>
<tr>
<td>7</td>
<td>23</td>
<td>19.05</td>
<td>10.8</td>
<td>6</td>
<td>5.1</td>
<td>136</td>
<td>5.7</td>
<td>4.7</td>
</tr>
<tr>
<td>8</td>
<td>22</td>
<td>19.05</td>
<td>10.8</td>
<td>6</td>
<td>4.6</td>
<td>134</td>
<td>5.2</td>
<td>4.2</td>
</tr>
<tr>
<td>9</td>
<td>22</td>
<td>19.05</td>
<td>11</td>
<td>6</td>
<td>4.6</td>
<td>134</td>
<td>5.1</td>
<td>4.2</td>
</tr>
<tr>
<td>10</td>
<td>22</td>
<td>19.05</td>
<td>12</td>
<td>6</td>
<td>4.5</td>
<td>133.5</td>
<td>4.9</td>
<td>4.1</td>
</tr>
<tr>
<td>11</td>
<td>22</td>
<td>19.05</td>
<td>10</td>
<td>6</td>
<td>4.8</td>
<td>134</td>
<td>5.4</td>
<td>4.4</td>
</tr>
<tr>
<td>12</td>
<td>22</td>
<td>19.05</td>
<td>9</td>
<td>6</td>
<td>5.0</td>
<td>134.5</td>
<td>5.7</td>
<td>4.5</td>
</tr>
<tr>
<td>13</td>
<td>22</td>
<td>19.05</td>
<td>8</td>
<td>6</td>
<td>5.3</td>
<td>135.5</td>
<td>6</td>
<td>4.8</td>
</tr>
<tr>
<td>14</td>
<td>24</td>
<td>19.05</td>
<td>12</td>
<td>6</td>
<td>5.4</td>
<td>137</td>
<td>6.2</td>
<td>4.8</td>
</tr>
<tr>
<td>15</td>
<td>24</td>
<td>20</td>
<td>12</td>
<td>6</td>
<td>5.1</td>
<td>136</td>
<td>5.7</td>
<td>4.6</td>
</tr>
<tr>
<td>16</td>
<td>24</td>
<td>20</td>
<td>10</td>
<td>6</td>
<td>5.4</td>
<td>136.5</td>
<td>6.4</td>
<td>4.9</td>
</tr>
<tr>
<td>17</td>
<td>24</td>
<td>21</td>
<td>10</td>
<td>6</td>
<td>5.0</td>
<td>135</td>
<td>5.7</td>
<td>4.5</td>
</tr>
<tr>
<td>18</td>
<td>25.4</td>
<td>21</td>
<td>10</td>
<td>6</td>
<td>5.8</td>
<td>138</td>
<td>6.8</td>
<td>5.3</td>
</tr>
<tr>
<td>19</td>
<td>25.4</td>
<td>21</td>
<td>8</td>
<td>8</td>
<td>5.6</td>
<td>140</td>
<td>6.6</td>
<td>5.1</td>
</tr>
<tr>
<td>20</td>
<td>22</td>
<td>19.05</td>
<td>8</td>
<td>8</td>
<td>4.7</td>
<td>136.5</td>
<td>5.2</td>
<td>4.3</td>
</tr>
<tr>
<td>21</td>
<td>23</td>
<td>19.05</td>
<td>8</td>
<td>8</td>
<td>5.1</td>
<td>138.5</td>
<td>5.9</td>
<td>4.7</td>
</tr>
<tr>
<td>22</td>
<td>25.4</td>
<td>22</td>
<td>8</td>
<td>8</td>
<td>5.2</td>
<td>138.5</td>
<td>6.0</td>
<td>4.7</td>
</tr>
<tr>
<td>23</td>
<td>25.4</td>
<td>22</td>
<td>7</td>
<td>8</td>
<td>5.5</td>
<td>139</td>
<td>6.5</td>
<td>5</td>
</tr>
<tr>
<td>24</td>
<td>27</td>
<td>23</td>
<td>7</td>
<td>8</td>
<td>6.1</td>
<td>141</td>
<td>7.1</td>
<td>5.6</td>
</tr>
<tr>
<td>25</td>
<td>27</td>
<td>23</td>
<td>8</td>
<td>8</td>
<td>5.7</td>
<td>140</td>
<td>6.7</td>
<td>5.1</td>
</tr>
<tr>
<td>26</td>
<td>27</td>
<td>24</td>
<td>8</td>
<td>8</td>
<td>5.2</td>
<td>138.5</td>
<td>6.0</td>
<td>4.7</td>
</tr>
</tbody>
</table>

According to the results seen in Table 3.4, the 8th, 16th and 22nd sets are selected for the manufacturing stage. These transducers are working below 7 kHz and their quality...
factors are below 5 which is an acceptable value for the purpose of the study. These transducers will be here after referred to as Set-8, Set-16 and Set-22, respectively.

The resonance frequencies of the transducers are calculated at 4.6 kHz, 5.4 kHz, 5.2 kHz for Set-8, Set-16 and Set-22, respectively, according to conductance of the transducer, as shown in Figure 3.10. TVR results are found parallel to conductance as shown in Figure 3.11. The transducers have a second resonance at about 12.5 kHz which increases the bandwidth of TVR.

![Figure 3.10 Conductance results of the transducers calculated by FEM](image1)

![Figure 3.11 TVR results of the transducers calculated by FEM](image2)

It can be seen in Figure 3.12 that maximum RVS values are over $-155 \, dB$ which is a good value for a transducer serving as a projector. Maximum RVS of transducers are
calculated at 5.4, 6.3 and 6.1 for Set-8, Set-16 and Set-22, respectively. These frequencies are higher than the frequencies where maximum TVRs are calculated.

![Figure 3.12 RVS calculated by FEM](image)

![Figure 3.13 Beam patterns at resonance frequencies of each transducer calculated by FEM](image)

Beamwidth of transducers are calculated at their own resonance frequencies. According to the beam pattern of transducers shown in Figure 3.13, transducers are found omni-directional at their own resonance frequencies. This is unexpected result and is discussed in Chapter 5. The values for the selected transducers are suitable for manufacturing and characterization. The next step is to manufacture and perform the acoustic tests.
CHAPTER 4

MANUFACTURING AND CHARACTERIZATION OF THE SELECTED TRANSDUCERS

4.1 Manufacturing Steps

Transducers are analyzed with some assumptions and simplifications. Glue, electrodes between ceramics and isolaters are omitted in the design but should be considered for the manufacturing. The housing which protects transducer parts from the environmental effects and the rubber coating on head mass and housing are to be considered as well. The electronic and electrical parts are also important in manufacturing of transducer. Housing and head mass without rubber coating and tail mass are seen in Figure 4.1.

![Figure 4.1 Head mass, tail mass and cylindrical housing](image)

Figure 4.1 Head mass, tail mass and cylindrical housing
Head masses and housings are covered by the neoprene rubber. This is an important step in the manufacturing because of the impermeability requirement of the transducers. Transducers operate underwater and they should be protected from water to keep the electrical system inner inside safe as well as to increase insulation resistance of the whole transducer against water. Head masses of the transducers with neoprene rubber cover are shown in Figure 4.2.

![Figure 4.2 Head masses covered by neoprene rubber and stress rods assembled](image)

Piezoceramics are stacked in parallel using a conductive epoxy glue. A 50 μm thick Cu-Be electrode is placed between two piezoceramics and they are glued together. A master is designed to align ceramics on top of each other in the stack as shown in Figure 4.3. Two insulators made from FR4 (glass fiber reinforced epoxy) material are used between piezoceramic stack and head and tail masses for electricity insulation.
After they are aligned and glued, they are squeezed with a bolt passing through the middle of it and then kept at 60 °C for 1 hour to cure the glue. Then, the stack is separated from the master. After this process, they are covered by outside epoxy-glass fiber cloth and inside polyurethane potting as shown in Figure 4.4 to keep the alignment of the stacks in case of cracking during working as well as to prevent electrical short-cut in case of contact with water.

Figure 4.4  Piezoceramic stack separated from master (left) and after covered by outside glass fiber and inside polyurethane (right)
After piezoceramic stacks are prepared, the main parts of the transducers are assembled such that piezoceramic stacks are placed between head and tail masses, and squeezed by stress rods tightened at 50 Nm torque as shown in Figure 4.5. A non-conductive epoxy-glue was applied to the contact surfaces between metal parts and stack before squeezing the rods.

Figure 4.5 Transducers after the electrical and mechanical parts assembled

Transducers are encapsulated by neoprene-coated cylindrical housings together with an electrical cable. They are all-glued together with a rubber-based glue and then kept at 60 °C for 1 hour to fully cure and seal the transducers. The finished product is shown in Figure 4.6.

Figure 4.6 A finished transducer ready for measurements
4.2 Characterization and Measurement

The characterization and experimental measurement steps of the transducers are performed at the facilities of the METEKȘAN Defense Industry. Open-water test and calibration facility is established on a floating pier at the Bilkent Lake. There is a positioning system on the platform with 2 tons lifting capacity. The facility has the capacity to measure acoustic parameters in the range from 1 to 100 kHz in the far field.

In the cylindrical pressure tank, acoustic measurements can be carried out up to 50 bars. Pressure tank is 7.5 m. in length and 2.5 m. in diameter. The tank has the capacity to make the acoustic measurements in the range from 10 to 100 kHz. Pressure tank and acoustic test pool facilities can be seen in Figure 4.7.

![Figure 4.7 Pressure tank and acoustic test pool](image)

The measurements of the transducers are performed in the acoustic test pool. The pool is 6 m. in length, 4 m. in width and 4 m. in depth. In the pool, there is a positioning system to precisely locate the transducers to be measured. Secondary methods are used to calculate the acoustic performance. A reference transducer is located exactly in front of the transducer to a distance of 2 m in depth and 2 m from the surface of the pool. In the TVR measurements, the sound generated by the transducer under alternating
voltage is measured by a reference transducer. TVR results are obtained in the range from 3 to 13 kHz with a step size of 0.1 kHz and compared in Figure 4.8.

The peak TVRs are measured at 4.7, 4.9 and 4.9 kHz for the transducer Sets 8, 16 and 22, respectively. Best TVR level is found in the Set-22 and TVR levels decrease with decreasing peak frequencies. Maximum TVRs are found as 132.3, 133.7 and 137.3 dB and 3 dB bandwidths are 1.2, 1.4 and 1.4 kHz for the Sets 8, 16 and 22, respectively. TVR does not fall below 125 dB for a long range of frequency (nearly from 4 to 12.5 kHz) because of the second modes at nearly 12.5 kHz (see Figure 4.8)

![Figure 4.8 TVR measurements of the selected transducers](image)

The conductance of the transducers is also measured in water for the same range as TVR and compared in Figure 4.9. The resonance frequencies are estimated to be 4.4, 4.7 and 4.9 kHz for the Sets 8, 16 and 22, respectively. The maximum TVRs are calculated at slightly higher frequencies due to some frequency dependent parameters as mentioned in Chapter 3.
On the contrary to the TVR measurements, the reference transducer is used as projector and fabricated transducers are used as hydrophones in the RVS measurements. The RVS results are measured in the same frequency range as TVR and compared in Figure 4.10. RVS results are found as -154.7 dB at 4.7 kHz, -159.1 dB at 4.9 kHz and -156.8 dB at 5 kHz for the Set-8, Set-16 and Set-22, respectively. RVS values are sufficient at resonance frequency but the effect of the second mode at 11.4 kHz is not observed here. Therefore, the decline in the RVS values is more sharply than the TVR values.
In the beam pattern measurements, the transducer is rotated horizontally from -90 degrees to 90 degrees according to its first position at intervals of 5 degrees. The sound waves are generated at the respective resonance frequency of each transducer and measured by a reference hydrophone. The differences in transducer gains from the zero angle is calculated and given in Figure 4.11. It can be seen that 3 dB beamwidths of the transducers are close to each other and 115, 100 and 110 degrees for the Set-8, Set-16 and Set-22, respectively.

Figure 4.11 Beam patterns of the transducers
CHAPTER 5

RESULTS AND DISCUSSION

Three transducers are determined in three design steps which are simple lumped parameter method, ECM and FEM, according to design criteria specified before the analysis and their acoustic performance parameters. After that, the selected transducers are manufactured and tested. In order to compare the results, for the manufactured transducers, ECM is also applied in addition to FEM. Acoustic performance parameters of the transducers obtained from design and experimental studies are compared to each other and the relative deviations for each one is determined and discussed.

5.1 Transducer Set-8

Conductance modelled with the design methods and experimental results are shown in Figure 5.1 and their comparison is given in Table 5.1. Resonance frequencies are calculated at reasonably good accuracy for all design methods. Increasing the detail of model constructed with design methods increase the accuracy of the calculated parameters as can be seen in the comparison of the design methods. ECM are sufficient in determining the first resonance frequencies but the second mode is not observed in these methods. The second resonance at about 11.3 kHz according to the measurement results is calculated at 12.3 kHz with FEM.
TVR modelled with the design methods and experimental results are shown in Figure 5.2 and their comparison is given in Table 5.2. The working frequency of the transducer is calculated by ECM with 8.5% deviation. Besides, the improved ECM and FEM are matched to great extent with experimental measurements. TVR results are generally in good agreement with experimental study. Maximum TVR values and the values in other frequencies calculated by design methods and experimental studies well-matched to each other. Because of the second mode of the transducer, a peak value is calculated at 12.3 kHz by FEM. On the other hand, quality factor calculations of the design methods are not good as in TVR. Best result is determined by FEM but with a 18% deviation.
RVS modelled with the design methods and experimental results are shown in Figure 5.3 and their comparison is given in Table 5.3. The RVS results are not in good agreement with the experimental measurements, which is mainly because of the calculation method used in the design methods. In the design methods, RVS is calculated by the reciprocity formula (Eq. 1.28) after TVR is calculated. On the contrary, in the experimental study, it is calculated by directly measuring the open circuit voltage of the transducer using a reference projector. For better results, the transducer can be analyzed as a hydrophone.
Figure 5.3 Comparison of the RVS results

Table 5.3 Comparison of the peak RVS frequency and the RVS results

<table>
<thead>
<tr>
<th></th>
<th>$f_{p-rvs}$</th>
<th>Relative deviation (%)</th>
<th>Max RVS</th>
<th>Relative deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECM (Basic)</td>
<td>5.7</td>
<td>21</td>
<td>-159</td>
<td>2.7</td>
</tr>
<tr>
<td>ECM (Improved)</td>
<td>5.4</td>
<td>15</td>
<td>-158.2</td>
<td>2.2</td>
</tr>
<tr>
<td>FEM</td>
<td>5.4</td>
<td>15</td>
<td>-149.7</td>
<td>3.2</td>
</tr>
<tr>
<td>Experimental</td>
<td>4.7</td>
<td>-</td>
<td>-154.7</td>
<td>-</td>
</tr>
</tbody>
</table>

Beam pattern is calculated only by FEM and compared to the experimental results as shown in Figure 5.4. Beam pattern calculations are performed at the resonance frequency of the transducer which is 4.6 kHz for FEM and 4.7 kHz for the measurement. The measured 3 dB beamwidth is about 115 degrees. On the other hand, it is calculated as omni-directional by FEM. TVR does not fall below 3 dB of the maximum TVR at the acoustic axis.

In Table 5.4, all results calculated by design methods and experimental measurements are compared.
Figure 5.4 Comparison of the beam patterns

Table 5.4 Comparison of the results

<table>
<thead>
<tr>
<th></th>
<th>ECM (Basic)</th>
<th>ECM (Improved)</th>
<th>FEM</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_n$ (kHz)</td>
<td>5.2</td>
<td>4.8</td>
<td>4.6</td>
<td>4.4</td>
</tr>
<tr>
<td>$f_p$ (TVR) (kHz)</td>
<td>5.1</td>
<td>4.8</td>
<td>4.6</td>
<td>4.7</td>
</tr>
<tr>
<td>$f_p$ (RVS) (kHz)</td>
<td>5.7</td>
<td>5.4</td>
<td>5.4</td>
<td>4.7</td>
</tr>
<tr>
<td>Bandwidth of TVR (kHz)</td>
<td>1.1</td>
<td>1</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>4.6</td>
<td>4.8</td>
<td>4.6</td>
<td>3.9</td>
</tr>
<tr>
<td>Max TVR (dB)</td>
<td>132.5</td>
<td>132.5</td>
<td>134</td>
<td>132.3</td>
</tr>
<tr>
<td>Max RVS (dB)</td>
<td>-159</td>
<td>-158.2</td>
<td>-149.7</td>
<td>-154.7</td>
</tr>
<tr>
<td>Beamwidth</td>
<td>-</td>
<td>-</td>
<td>Omni-directional</td>
<td>115°</td>
</tr>
</tbody>
</table>

5.2 Transducer Set-16

Conductance modelled with the design methods and experimental results are shown in Figure 5.5 and their comparison is given in Table 5.5. Resonance frequencies are calculated at 6.1, 5.5 and 5.4 kHz by ECM (basic), ECM (improved) and FEM, respectively. The actual value is experimentally determined at 4.6 kHz.
Figure 5.5 Comparison of the conductance results

Table 5.5 Comparison of the resonance frequencies

<table>
<thead>
<tr>
<th>Method</th>
<th>$f_n$</th>
<th>Relative deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECM (Basic)</td>
<td>6.1</td>
<td>41</td>
</tr>
<tr>
<td>ECM (Improved)</td>
<td>5.5</td>
<td>20</td>
</tr>
<tr>
<td>FEM</td>
<td>5.4</td>
<td>17</td>
</tr>
<tr>
<td>Experimental</td>
<td>4.6</td>
<td>-</td>
</tr>
</tbody>
</table>

TVR modelled with the design methods and experimental results are shown in Figure 5.6 and their comparisons are given in Table 5.6. The peak frequency of the transducer is calculated by ECM with 24 percent deviation. The improved method gives better results with a deviation of 12 percent. The working frequency is calculated in FEM with a deviation of 10 percent. TVR results are generally in good agreement with experimental study. Maximum TVR is calculated by design methods with about 3.4 deviations and TVR values in other frequencies matched with experimental study at very good accuracy. On the contrary to the first transducer, better results are obtained in quality factor calculations as shown in Table 5.6. ECM without passive elements give better results in quality factor calculations compared to the ECM with passive
elements. This may be result of inaccuracy of the ECM (basic) in the peak frequency which is much higher than the experimental value.

![Graph](image.png)

**Figure 5.6** Comparison of the TVR results

<table>
<thead>
<tr>
<th></th>
<th>( f_p )</th>
<th>Deviation (%)</th>
<th>Max TVR</th>
<th>Deviation (%)</th>
<th>( Q_m )</th>
<th>Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECM (Basic)</td>
<td>6.1</td>
<td>24</td>
<td>136.8</td>
<td>3.6</td>
<td>3.8</td>
<td>8.5</td>
</tr>
<tr>
<td>ECM (Improved)</td>
<td>5.5</td>
<td>12</td>
<td>136.7</td>
<td>3.6</td>
<td>5</td>
<td>42</td>
</tr>
<tr>
<td>FEM</td>
<td>5.4</td>
<td>10</td>
<td>136.5</td>
<td>3.4</td>
<td>3.6</td>
<td>2.8</td>
</tr>
<tr>
<td>Experimental</td>
<td>4.9</td>
<td>-</td>
<td>133.7</td>
<td>-</td>
<td>3.5</td>
<td>-</td>
</tr>
</tbody>
</table>

RVS modelled with the design methods and experimental results are shown in Figure 5.7 and their comparison is given in Table 5.7. Peak frequencies from RVS plots are calculated at 7.1, 6.6 and 6.3 kHz for ECM (basic), ECM (improved) and FEM, respectively. It is measured at 4.9 kHz experimentally. Max RVS values and the RVS trends are in good agreement in all design methods.
Beam pattern is not well-matched with the experimental measurements. The calculation results indicate that the transducer is omni-directional but experimental result show a beamwidth of about 100 degrees as shown in Figure 5.8.

In Table 5.8, all the results calculated by design methods and experimental measurement results for the transducer are given.
Figure 5.8 Comparison of the beam pattern

Table 5.8 Comparison of the results

<table>
<thead>
<tr>
<th></th>
<th>ECM (Basic)</th>
<th>ECM (Improved)</th>
<th>FEM</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_n$ (kHz)</td>
<td>6.1</td>
<td>5.5</td>
<td>5.4</td>
<td>4.6</td>
</tr>
<tr>
<td>$f_p$ (TVR) (kHz)</td>
<td>6.1</td>
<td>5.5</td>
<td>5.4</td>
<td>4.9</td>
</tr>
<tr>
<td>$f_p$ (RVS) (kHz)</td>
<td>7.1</td>
<td>6.6</td>
<td>6.3</td>
<td>4.9</td>
</tr>
<tr>
<td>Bandwidth (kHz)</td>
<td>1.6</td>
<td>1.1</td>
<td>1.5</td>
<td>1.4</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>3.8</td>
<td>5</td>
<td>3.6</td>
<td>3.5</td>
</tr>
<tr>
<td>Max TVR (dB)</td>
<td>136.8</td>
<td>136.7</td>
<td>136.5</td>
<td>133.7</td>
</tr>
<tr>
<td>Max RVS (dB)</td>
<td>-161.6</td>
<td>-160.8</td>
<td>-153.8</td>
<td>-159.1</td>
</tr>
<tr>
<td>Beamwidth</td>
<td>-</td>
<td>-</td>
<td>Omni-directional</td>
<td>100°</td>
</tr>
</tbody>
</table>

5.3 Transducer Set-22

Conductance modelled with the design methods and experimental results are shown in Figure 5.9 and their comparison is given in Table 5.9. Resonance frequency calculated by design methods are in good agreement with the experimental result. The deviation
from measurement is 6 percent for FEM and improved ECM. The ECM without passive parts give about 16 percent error by comparing with the experimental study.

Figure 5.9 Comparison of the conductance results

Table 5.9 Comparison of the resonance frequencies

<table>
<thead>
<tr>
<th></th>
<th>( f_n )</th>
<th>Relative deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECM (Basic)</td>
<td>5.7</td>
<td>16</td>
</tr>
<tr>
<td>ECM (Improved)</td>
<td>5.2</td>
<td>6</td>
</tr>
<tr>
<td>FEM</td>
<td>5.2</td>
<td>6</td>
</tr>
<tr>
<td>Experimental</td>
<td>4.9</td>
<td>-</td>
</tr>
</tbody>
</table>

TVR modelled with the design methods and experimental results are shown in Figure 5.10 and their comparison is given in Table 5.10. Resonance frequency and working frequency of the transducer is calculated and measured at the same frequency. Therefore, the deviations from measurements in TVR are same as in conductance. Both of working and resonance frequencies are in good agreement with measurement. \( f_p \) from max TVR is calculated at 5.7, 5.2, 5.2 kHz by ECM (basic), ECM (improved) and finite element methods, respectively. On the other hand, it is experimentally
measured at 4.9 kHz. The values are closed to each other. Furthermore, max TVR values calculated by design methods are nearly the same as the measurement results. Quality factor calculated with FEM is also closed to the measurement.

![Figure 5.10 Comparison of the TVR results](image)

**Table 5.10 Comparison of the TVR results**

<table>
<thead>
<tr>
<th></th>
<th>$f_p$</th>
<th>Deviation (%)</th>
<th>Max TVR</th>
<th>Deviation (%)</th>
<th>$Q_m$</th>
<th>Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECM (Basic)</td>
<td>5.7</td>
<td>16</td>
<td>137.2</td>
<td>0.07</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>ECM (Improved)</td>
<td>5.2</td>
<td>6</td>
<td>137.7</td>
<td>0.2</td>
<td>4.7</td>
<td>34</td>
</tr>
<tr>
<td>FEM</td>
<td>5.2</td>
<td>6</td>
<td>138.6</td>
<td>0.9</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>Experimental</td>
<td>4.9</td>
<td>-</td>
<td>137.3</td>
<td>-</td>
<td>3.5</td>
<td>-</td>
</tr>
</tbody>
</table>

RVS modelled with the design methods and experimental results are shown in Figure 5.11 and their comparison is given in Table 5.11. The max RVS values calculated by FEM and experimental study are closed to each other but the peak frequencies are not in good agreement. On the other hand, the results calculated by design methods match well with the experimental results when whole frequency range is considered.
Figure 5.11 Comparison of the RVS results

Table 5.11 Comparison of the peak RVS frequency and the RVS results

<table>
<thead>
<tr>
<th></th>
<th>$f_{p-rvs}$</th>
<th>Relative deviation (%)</th>
<th>Max RVS</th>
<th>Relative deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECM (Basic)</td>
<td>6.6</td>
<td>32</td>
<td>-163.2</td>
<td>4</td>
</tr>
<tr>
<td>ECM (Improved)</td>
<td>6</td>
<td>20</td>
<td>-162.2</td>
<td>3.4</td>
</tr>
<tr>
<td>FEM</td>
<td>6.1</td>
<td>22</td>
<td>-154.9</td>
<td>1.2</td>
</tr>
<tr>
<td>Experimental</td>
<td>5</td>
<td>-</td>
<td>-156.8</td>
<td>-</td>
</tr>
</tbody>
</table>

As for all transducers, beam is calculated as omni-directional by FEM. However, it has a beamwidth about 110 degrees as shown in Figure 5.12.

The results discussed in this part for the Set-22 transducer is given in Table 5.12.
Here, it should be mentioned that the peak frequencies in the TVR and RVS plots are nearly the same in the measurements while, in the design methods, the peak of the RVS is calculated at higher frequency than of the TVR one. In the design methods, RVS is calculated by reciprocity (Eq. 1.28) formula after TVR is calculated. In the actual measurements, however, RVS is calculated directly by using it as a hydrophone. The same situation is also observed by Miyama et al. [36]. They studied a Tonpilz

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ECM (Basic)</th>
<th>ECM (Improved)</th>
<th>FEM</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_n$ (kHz)</td>
<td>5.7</td>
<td>5.2</td>
<td>5.2</td>
<td>4.9</td>
</tr>
<tr>
<td>$f_p$ (TVR) (kHz)</td>
<td>5.7</td>
<td>5.2</td>
<td>5.2</td>
<td>4.9</td>
</tr>
<tr>
<td>$f_p$ (RVS) (kHz)</td>
<td>6.6</td>
<td>6</td>
<td>6.1</td>
<td>5</td>
</tr>
<tr>
<td>Bandwidth (kHz)</td>
<td>1.4</td>
<td>1.1</td>
<td>1.3</td>
<td>1.4</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>4</td>
<td>4.7</td>
<td>4</td>
<td>3.5</td>
</tr>
<tr>
<td>Max TVR (dB)</td>
<td>137.2</td>
<td>137.7</td>
<td>138.6</td>
<td>137.3</td>
</tr>
<tr>
<td>Max RVS (dB)</td>
<td>-163.2</td>
<td>-162.2</td>
<td>-154.9</td>
<td>-156.8</td>
</tr>
<tr>
<td>Beamwidth</td>
<td>-</td>
<td>-</td>
<td>Omni-directional</td>
<td>110º</td>
</tr>
</tbody>
</table>

![Figure 5.12 Comparison of the beam pattern](image-url)
transducer and calculated TVR and RVS by ECM and compared them to the actual measurement results. It can be seen from Figure 5.13 that peaks of the TVR and RVS are nearly the same for the measurement (dashed lines) while there is approximately %11 deviation between them when calculated by ECM (solid lines).

Figure 5.13 TVR and RVS plots studied in reference [36]

The other important detail is the sharp decrease after the peak frequency is exceeded in the RVS results (e.g., Figure 5.11) as compared to the TVR results (e.g., Figure 5.10). The effect of second mode at nearly 12.4 kHz is not observed in the RVS results while it broadens the bandwidth in the TVR plot. The same effect is also observed in literature [54]. As shown in Figure 5.14, the TVR has a second peak at about 9 kHz which broadens the bandwidth while the RVS rapidly falls down after the first (i.e., resonance) peak.

Figure 5.14 TVR and RVS plots studied in Reference [54]
In order to detail possible reasons for this situation, the first and second modes of the transducers are investigated. The displacement responses of the transducers at first and second resonance frequencies are shown in Figure 5.15 and Figure 5.16. The first mode is the piston motion where the transducer parts moving forward and backward together in the polarization direction. In the second mode, corners of the head mass moves in opposite direction to the centre, as inside and outside. This is the flexural mode of the head mass which in general adversely affects the transducer performance. However, there are some reports [40] using the flexural mode to broaden the TVR bandwidth, as is the case in our study. On the other hand, this flexural mode does not generate high enough voltages in piezoceramics to increase RVS in that frequency. Therefore, it may be the reason for the effect observed in TVR but not in RVS.

Figure 5.15 First mode (piston mode) of the transducer Set-16 at 5.4 kHz

Figure 5.16 Second mode (flexural mode) of the transducer Set-16 at 12.4 kHz
The head mass and all the other parts are modelled in the ECM by their physical properties such as stiffness and mass. Their shapes and dimensions can not be modelled in these methods. Therefore, velocity of the head mass is assumed to be uniform in ECM. As a result, the second mode which is the flexural mode of head mass can not be observed in the ECM.

There are some simplifications and assumptions made in the design methods which are the reasons for the calculated deviations. While designing the transducer, the parts of the transducer always presumed to be uniformly connected to each other which is not totally possible in practice. In addition, the in-water measurement environment is assumed to be ideal in the design methods but it is not possible in the real measurement conditions (e.g., turbidity, temperature gradient, salinity etc). The rubber coating of the head mass is neglected in the design stage which can severely affect impedance, resonance frequency and beam pattern. Prestress induced in transducer due to tightening torque by means of a stress bolt is also not modelled in the design methods. The torque creates a compression on the ceramics which adjusts the natural frequency and resulting acoustic performances. In the FEM, the passive parts such as electrodes, glue and isolators are not modelled. The mechanical and electrical efficiencies can not be accurately modelled in the ECM as well.

There is nearly the same shift always between improved ECM and basic ECM. Improved ECM is at left of the basic ECM for all calculations. This is because of the parameters such as glue, electrodes and isolators which are added to the piezoceramics as parallel capacitors.

It should be indicated that the beampattern of the transducer is not calculated well. This is mainly because the rubber coating is not modelled in the design methods but directly affects the beamwidth of the transducer. The deviation may be because of the environmental conditions which is assumed ideal in the FEM. The constraints of the design methods and comparison to measurement can be seen in Table 5.13.
Table 5.13 Comparison of constraints of the models and the actual measurement

<table>
<thead>
<tr>
<th></th>
<th>ECM (Basic)</th>
<th>ECM (Improved)</th>
<th>FEM</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main parts (Head, tail, stress rod, piezoceramics)</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Passive parts (Glue, electrodes, isolators)</strong></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Neoprene rubber</strong></td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Water</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>3-D modelling of parts</strong></td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Uniform velocity assumption for head mass</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Prestress occurred in stress rod</strong></td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>RVS calculation</strong></td>
<td>From TVR (By reciprocity formula)</td>
<td>From TVR (By reciprocity formula)</td>
<td>From TVR (By reciprocity formula)</td>
<td>Directly measured</td>
</tr>
</tbody>
</table>

Table 5.14 shows the relative deviations of the design methods when compared to the measurement results. Among the design methods used in this thesis, FEM which has minimum relative deviations, is the best option for designing a transducer. It is the most detailed model as seen in Table 5.13.

The accuracy can be improved by modelling the skipped parts such as glue, electrodes, isolators or neoprene rubber coating. On the other hand, in order to get more accurate results in the RVS calculation, rather than using a reciprocity formula, transducer can be analyzed as hydrophone and RVS can be directly measured. Also, if the prestress occurred in stress rod can be modelled in the FEM, the deviations will decrease to quite small levels.

In the ECM, there are more simplifications because of the nature of the method. Nevertheless, this method can also be improved by modelling neoprene rubber or prestress somehow in the circuit. This may decrease the relative deviations in the ECM.

Among the design methods used in the thesis, FEM is the one which gives the most accurate results. By considering the deviations in the results obtained by FEM, Set-8 and Set-22 are better than Set-16 and the deviations are closed to each other. Although
it is difficult to choose one of the transducers, it can be said that the transducer Set-22 is the best transducer regarding the operating frequency, quality factor and TVR level which are the most important performance parameters in this study.

However, it should be stated that design, characterization and measurements of all the transducers are accomplished successfully and the deviations in the results obtained by the design methods are acceptable and consistent with similar results obtained from Tonpilz-type transducers reported in the literature. Also, all of the transducers are operating below 5 kHz which is better than the desired frequency and the performances of all the transducers are quite well.

Table 5.14 The relative deviations (%) calculated in the design methods

<table>
<thead>
<tr>
<th></th>
<th>ECM (Basic)</th>
<th>ECM (Improved)</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set-8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_n$</td>
<td>18</td>
<td>9</td>
<td>4.5</td>
</tr>
<tr>
<td>$f_p$ (TVR)</td>
<td>8.5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$f_p$ (RVS)</td>
<td>21</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>8.3</td>
<td>16.6</td>
<td>16.6</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>18</td>
<td>23</td>
<td>18</td>
</tr>
<tr>
<td>Max TVR</td>
<td>0.4</td>
<td>0.4</td>
<td>1.5</td>
</tr>
<tr>
<td>Max RVS</td>
<td>2.7</td>
<td>2.2</td>
<td>3.2</td>
</tr>
<tr>
<td>Set-16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_n$</td>
<td>41</td>
<td>20</td>
<td>17</td>
</tr>
<tr>
<td>$f_p$ (TVR)</td>
<td>24</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>$f_p$ (RVS)</td>
<td>45</td>
<td>34</td>
<td>28</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>14.3</td>
<td>21.4</td>
<td>7.1</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>8.5</td>
<td>42</td>
<td>2.8</td>
</tr>
<tr>
<td>Max TVR</td>
<td>3.6</td>
<td>3.6</td>
<td>3.4</td>
</tr>
<tr>
<td>Max RVS</td>
<td>1.5</td>
<td>1</td>
<td>3.3</td>
</tr>
<tr>
<td>Set-22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_n$</td>
<td>16</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>$f_p$ (TVR)</td>
<td>16</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>$f_p$ (RVS)</td>
<td>32</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>0</td>
<td>21.4</td>
<td>7.1</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>14</td>
<td>34</td>
<td>14</td>
</tr>
<tr>
<td>Max TVR</td>
<td>0.07</td>
<td>0.2</td>
<td>0.9</td>
</tr>
<tr>
<td>Max RVS</td>
<td>4</td>
<td>3.4</td>
<td>1.2</td>
</tr>
</tbody>
</table>
CHAPTER 6

CONCLUSION

The low frequency transducers are important for underwater sonar technologies for the defense industry. In order to decrease the working frequency of the transducer, the dimensions of the transducer are mostly required to be increased. Because of that, the low frequency transducers performing suitable performance parameters with minimum dimensions is an important challenge in the sonar studies. In this thesis, low frequency transducers with acceptable bandwidth in restricted dimensions are designed, manufactured and measured.

First of all, design criteria are specified for the study. The transducers are required to operate below 7 kHz with a minimum TVR of 130 dB. Maximum mechanical quality factor of the transducer is 5. The maximum RVS of the transducers should not be under -160 dB and the 100 degrees beamwidth is sufficient.

After the design criteria are specified, the design methods are clearly introduced before applying to the study. Simple lumped parameter method is the most basic modelling technique skipping most of the details of the transducer. The method only involves head and tail masses, and stiffness of the piezoceramics. This method is used only to determine the initial parameters according to the design requirements.

After the rough dimensions are obtained, the ECM which is more convenient including more details of a transducer, is used. In this method, the transducer is modelled as the electrical circuit. Two different ECM, namely improved and basic, are used. In the basic ECM, the main parts of the transducer such as head mass, tail mass, piezoceramic stack and stress rod are modelled. In the improved ECM, in addition to the main parts, passive elements such as glue, isolators and electrodes are also modelled in the circuit. In the ECM, only head and tail masses are used as variables in optimization. A transducer operating at 5.8 kHz with maximum TVR of 137.5 dB is calculated in this step.
Because the ECM only allows to model the transducer parts with their mechanical properties and physical parameters, the final form of the transducer is determined in the FEM. A transducer is designed in SOLIDWORKS according to the parameters obtained in the ECM. Then, the transducer is analyzed in COMSOL for many times by changing the number and dimensions of piezoceramics and corresponding TVR results are observed. When considering TVR results, three transducers, namely Set-8, Set-16 and Set-22, operating at 4.6 kHz, 5.4 kHz and 5.2 kHz with mechanical quality factors of 4.8, 3.6 and 4 are selected as the best options. The maximum RVS values are calculated at 5.4 kHz as -149.7 dB, at 6.3 kHz as -153.8 dB and at 6.1 kHz as -154.9 dB, respectively, for transducers Set-8, Set-16 and Set-22. According to the beampattern calculation, transducer is determined as omni-directional. These values are found sufficient when considering the design criteria and the transducers are manufactured.

The acoustic parameters of the transducers are characterized at the Meteksan Defense Industry Inc. (Ankara). Operating frequencies of the transducers are measured at 4.4 kHz, 4.7 kHz and 4.9 kHz with mechanical quality factors of 3.9, 3.5 and 3.5 for Set-8, Set-16 and Set-22, respectively. The maximum RVS values are found as -154.7 dB at 4.7 kHz, -159.1 dB at 4.9 kHz and -156.8 dB at 5 kHz for the Set-8, Set-16 and Set-22, respectively. According to the beampattern measurements, 3 dB beamwidth of transducers are measured to be 115°, 100° and 110° for the transducers Set-8, Set-16 and Set-22, respectively. These results are very promising for the aim of the study.

For the last part of the study, the results obtained from experiments and design steps are compared with each other. The degree of details in the design techniques improves the reliability of the method. According to the accuracy of the study, FEM is the best option among the design methods. On the other hand, the solution time is much more than the ECM. Therefore, before FEM, using ECM can decrease the amount of time of the study.

Finally, three low frequency Tonpilz transducers operating well below 7 kHz and performing sufficient acoustic performance for underwater applications are successfully designed, manufactured and characterized. Furthermore, the comparison of the results obtained from the design methods and experimental measurements show
that calculated acoustic parameters are fairly good, although there are some deficiencies to be improved in the design methods.

Most important parameters of the Tonpilz-type transducers are operating frequency, TVR level and bandwidth. Further studies can be performed to decrease the resonance and working frequency of the transducer. The TVR levels of the transducers are good and the bandwidths are sufficient but they can also be improved by using specific techniques available in the literature such as using void head mass or using flexural resonance. Additionally, in order to increase the accuracy of the design methods, the level of the details may be improved. For example, for the FEM, the prestress which occurs in the stress bolt could be added to the system. The passive elements such as glue, electrodes and isolators which are not modelled in FEM can be added to the system as well. Beampattern calculation should be improved because it is the weakest point of the FEM. In future works, the rubber coating on head mass may be modelled in the transducer models in order to improve the beampattern results.
REFERENCES


APPENDICES

Appendix A: Radiation Impedance Calculations

Appendix B: Piezoelectric Material Properties

Appendix C: Matlab Code for Equivalent Circuit Method
Appendix A – Radiation Impedance Calculations

Radiation impedance is an important parameter in underwater acoustic transducer design and can be defined as the ratio of the force exerted to the medium by the vibrating surface to the normal velocity of the surface [12]. It calculates how much acoustic power is generated in the medium according to the given mechanical vibration. Radiation impedance can be expressed as follows [56]:

\[
Z_r = R_r + jX_r = \frac{j \rho c k}{2\pi} \int \int S_0 \int S (e^{-jkr}/r) dSdS_0 = \rho c S \left[R_r \left(kS^{\frac{1}{2}}\right) + jX_r \left(kS^{\frac{1}{2}}\right)\right] 
\]

\[\text{(A.1)}\]

where \(Z_r\) is radiation impedance, \(R_r\) is radiation resistance and \(X_r\) is radiation reactance. Radiation resistance and radiation reactance are the real and imaginary parts of the radiation impedance respectively. In the equation, \(\rho c\) is characteristic impedance, \(k\) is wave number and \(S\) is area of the surface. This equation has reduced by Chetaev [55] to an expression which can be written after a simple transformation as follows [56]:

\[
R_r(v; \kappa) = 1 - \left(\frac{2}{\pi v^2}\right)[1 + \cos(vq) + vq \sin(vq) - \cos(v\beta) - \cos(v/\beta)] + \left(\frac{2}{\pi}\right) \beta \text{I}_1(v; \kappa) + \text{I}_1(v; 1/\kappa)/\beta
\]

\[\text{(A.2)}\]

\[
X_r(v; \kappa) = \left(\frac{2}{\pi v^2}\right)[\sin(vq) + vq \cos(vq) + v(\beta + 1/\beta) - \sin(v\beta) - \sin(v/\beta)] - \left(\frac{2}{\pi}\right) \beta \text{I}_2(v; \kappa) + \text{I}_2(v; 1/\kappa)/\beta
\]

\[\text{(A.2)}\]

where \(v = kS^{\frac{1}{2}}\), \(\kappa\) is aspect ratio of the surface, \(\beta = R^2\) and \(q = (R + 1/R)^{\frac{1}{2}}\).

\[
\text{I}_3(v; \xi) = \int_{\xi^{-1/2}}^{(\xi+1/\xi)^{1/2}} (1 - 1/\xi t^2)^{1/2} \cos(vt) \sin(vt) dt
\]

\[\text{(A.3)}\]
Here, the subscripts 1 and 2 refer to cosine and sine functions and $\xi$ stands for $\kappa$ or $1/\kappa$.

Knowing the aspect ratio $\kappa=1$ for square active surface, normalized radiation impedance terms are calculated with respect to a dimensionless number $ka$ as shown in Figure A.1. Here, $A$ is area of active surface and $a$ is a side length.

![Figure A.1](image)

**Figure A.1** Normalized radiation impedance with respect to $ka$
Appendix B – Piezoelectric Material Properties

Piezoelectric ceramics are mathematically described with linear equations between physical properties which are stress, $\sigma$, strain, $\varepsilon$, electric field, $E$ and electric displacement $D$. They are all position and time dependent and also they are assumed to be working in adiabatic conditions so temperature and entropy variables can be omitted. Electromechanical equations describing a linear piezoelectric are [1]:

\[
\varepsilon_i = s_{ij}^E \sigma_j + d_{mi}^t E_m
\]  \hspace{1cm} (B.1)

\[
D_m = d_{mi} \sigma_i + \varepsilon_{mk}^\sigma E_k
\]  \hspace{1cm} (B.2)

where, indexes $i, j, k = 1, 2, 3…, 6$ and $m, k = 1, 2, 3$ referring different directions in the material coordinate system represented in Figure B.1. There are many coefficients to solve, however for permanently polarized piezo ceramics some of the coefficients are zero or related to each other.

![Material coordinate system](image)

**Figure B.1** Material coordinate system

Electromechanical equation can be written in matrix notations as [1]:

\[
\begin{bmatrix}
\varepsilon_i \\
D_m
\end{bmatrix} =
\begin{bmatrix}
s_{ij}^E & d_{mi}^t \\
d_{mi} & \varepsilon_{mk}^\sigma
\end{bmatrix}
\begin{bmatrix}
\sigma_j \\
E_m
\end{bmatrix}
\]
\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6 
\end{bmatrix} = \begin{bmatrix}
S_{11}^E & S_{12}^E & S_{13}^E & 0 & 0 & 0 \\
S_{12}^E & S_{11}^E & S_{13}^E & 0 & 0 & 0 \\
S_{13}^E & S_{11}^E & S_{33}^E & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44}^E & 0 & 0 \\
0 & 0 & 0 & 0 & S_{44}^E & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}^E 
\end{bmatrix} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6 
\end{bmatrix}
\] (B.3)

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3 
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 & d_{31} \\
0 & 0 & 0 & 0 & d_{31} \\
d_{31} & d_{31} & d_{33} & 0 & 0 
\end{bmatrix} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6 
\end{bmatrix}
\] (B.4)

For the equations above:

\[
s_{11} = s_{22}, s_{13} = s_{31} = s_{32} = s_{23}, s_{12} = s_{21}, s_{44} = s_{55}, s_{66} = 2(s_{11} - s_{12})
\] (B.5)

\[
d_{31} = d_{32}, d_{15} = d_{24}
\] (B.6)

\[
\varepsilon_{11}^\sigma = \varepsilon_{22}^\sigma
\] (B.7)

where \(\varepsilon_4, \varepsilon_5, \varepsilon_6\) are shear strains and \(\sigma_4, \sigma_5, \sigma_6\) are shear stresses. These equations are also used in different pairs according to choose of independent variable depending on application [1].

\[
\sigma_i = c_{ij}^E \varepsilon_j - e_{ml}^E E_m
\] (B.8)
\[ D_m = e_{mi} \varepsilon_i + \varepsilon^e_{mk} E_k \]  
(B.9)

\[ \varepsilon_i = s^D_{ij} \sigma_j + g^r_{mi} D_m \]  
(B.10)

\[ E_m = -g_{mi} \sigma_i + \beta^\sigma_{mk} D_k \]  
(B.11)

\[ \sigma_i = c^D_{ij} \varepsilon_j - h^r_{mi} D_m \]  
(B.12)

\[ E_m = -h_{mi} S_i + \beta^e_{mk} D_k \]  
(B.13)
Appendix C – Matlab Code for Equivalent Circuit Method

Main Program

% Head mass
Mh = 0.46; % head mass, kg
a = 0.096; % edge length of active surface, m
Ah = a^2; % Area of active surface, m^2

% Tail mass
Mt = 1.84; % Tail mass, kg

% Piezoceramics

d2=24; % Outer radius of piezoceramics, m
d1=20; % Inner radius of piezoceramics, m
Ac = pi*(d2^2-d1^2)*10^-6; % Area of piezoceramics, m^2
n = 6; % number of piezoceramics in stack
m = 10e-3; % thickness of a piezoceramic, m
rho_c = 7500; % Density [kg/m^3]
Mcs = n*Ac*t*rho_c; % mass of piezoceramic stack, kg
d33 = 289*10^-12; % Piezoelectric charge constant, C/N
e33T = 1300*8.85e-12; % Permittivity coefficient at constant stress, F/m
s33E = 1.55*10^-11; % Elastic compliance, m^2/N
ek33 = d33/((s33E*e33T)^0.5); % Electromechanical coupling coefficient
kcs = Ac/(n*t*s33E); % stiffness of ceramic stack
Ccs = 1./kcs; % capacitance of ceramic stack

% Glue

Ag=Ac; % area of glue
Yg=3.5e9; % elasticity of glue
kg=Ag*Yg/(50e^-6); % stiffness of glue
Cg=(1/kg)*16; % Capacitance of glue

% Electrodes

Ye=110e9; % elasticity of electrodes
Ae=Ac; % area of electrodes
ke=Ae.*Ye/(50e^-6); % stiffness of electrodes
Ce=(1/ke)*7; % capacitance of electrodes

% Insulators

dii=36e-3; % inner diameter of insulators
dio=51e-3; % outer diameter of insulators
Ai=pi.*(dio^2-dii^2)/4; % Area of insulators
Yi=24e9; % Elasticity of insulators
ki=Ai.*Yi/(5e^-4); % Stiffness of insulators
Ci=(1/ki)*2; % Capacitance of insulators
C1=Ccs+Cg+Ce+Ci; % Total compliance of stack, glue, electrodes and insulators
% Stress rod

dsr = 12e-3; % Diameter of Stress rod, m
rho_sr = 7850; % Density of stress rod, kg/m^3
Ysr = 205e9; % Modulus of Elasticity of stress rod, Pa
lsr = 107e-3; % Length of stress rod, m
ksr = (Ysr*pi*(dsr/2)^2/(lsr)); % Stiffness of stress rod, Pa
Csr = 1/ksr; % Capacitance of stress rod

% Harmonic analysis

x=1;
for f = 3e3:100:13e3; % Frequency, Hz
w = 2.*pi.*f; % Angular Frequency
rho_w = 1000; % Density of acoustic medium, kg/m^3
C = 1500; % Speed of sound in acoustic medium, m/s
k = w/c; % Wave number

% Calculation of radiation terms

v = k.*a;
fun1 = @(t,v) (1-1./(t.^2)).^(1/2).*cos(v.*t);
I1 = integral(@(t)fun1(t,v),1,sqrt(2));
fun2 = @(t,v) (1-1./(t.^2)).^(1/2).*sin(v.*t);
I2 = integral(@(t)fun2(t,v),1,sqrt(2));
p = 1;
q = sqrt(2);
Rr(x) = c*rho_w*Ah*(1 - (2/(pi*v^2))*(1+cos(v*q) + v*q*sin(v*q) - cos(v*p) - cos(v/p)) + (2/pi)*(p*I1 + I1/p)); % Radiation resistance for square head mass
Xr(x) = c*rho_w*Ah*((2/(pi*v^2))*(sin(v*q) - v*q*cos(v*q) + v*(p+1/p) - sin(v*p) - sin(v/p)) - (2/pi)*(p*I2 + I2/p)); % Radiation reactance for square head mass
Mr(x) = Xr(x)/(f.*2.*pi); % Radiation mass for square head mass
K(x) = k;
W(x) = w;
F(x) = f;
Vv(x) = v;

% Equivalent masses

M1 = Mr(x) + Mh + Mcs/2; % Equivalent mass 1
M2 = Mt + Mcs/2; % Equivalent mass 2

% Equivalent compliance and resistances

Cm = 1/(1/Csr + 1/C1); % Total capacitance
R = Rr(x);

% Transformer ratio

N = d33*Ac/(t*s33E); % Transformer Turns Ratio
\[
B = \frac{1}{j \omega C_m} - j \omega M_{cs} R_2 + j \omega M_{cs} - j \omega M_2;
\]

\[
\text{total impedance}
\]

%Input voltage

\[
V = [N; 0];
\]

%Current through head mass

\[
u = \text{inv}(B) \cdot V;
\]

%Output velocity

\[
U_h = u(2);
\]

%Velocity of head mass

\[
u_h(x) = U_h;
\]

%Calculation of TVR

\[
p = W \cdot \rho_w \cdot u_h \cdot (a \cdot a) / (2 \cdot \pi); \quad \text{%Pressure in distance of 1 m}
\]

\[
\text{TVR} = 20 \log_{10} (\text{abs}(p) / (\sqrt{2} \cdot 1 \cdot 10^{-6}))
\]

\[
fo = F(\text{find(TVR == max(TVR), 1, 'last'))}; \quad \text{%Peak frequency}
\]

\[
\text{plot}(F, \text{TVR}); \quad \text{%TVR graph}
\]

Integral Calculation

\[
x = 1;
\]

for \( v = 1:0.1:10; \)

\[
\text{fun1} = @(t, v) (1 - 1/(t^2))^{1/2} \cdot \cos(v \cdot t); \quad \text{fun2} = @(t, v) (1 - 1/(t^2))^{1/2} \cdot \sin(v \cdot t); \quad \text{I1 = integral(\theta(t) \cdot \text{fun1}(t, v), 1, \sqrt{2});}
\]

\[
\text{I2 = integral(\theta(t) \cdot \text{fun2}(t, v), 1, \sqrt{2});}
\]

\[
p = 1; \quad q = \sqrt{2};
\]

\[
R(x) = 1 - (2/(\pi \cdot v^2)) \cdot (1 + \cos(v \cdot q) + v \cdot q \cdot \sin(v \cdot q) - \cos(v \cdot p) - \cos(v/p)) + (2/\pi) \cdot (p \cdot I1 + I1/p)
\]

\[
X(x) = (2/(\pi \cdot v^2)) \cdot (\sin(v \cdot q) - v \cdot q \cdot \cos(v \cdot q) + v \cdot (p+1/p) - \sin(v \cdot p) - \sin(v/p)) - (2/\pi) \cdot (p \cdot I2 + I2/p)
\]

\[
V(x) = v
\]

\[
x = x + 1;
\]

end
CURRICULUM VITAE

PERSONAL INFORMATION

Name Surname : POLAT KURT
Date of Birth : 20/04/1990
Phone : 0312 906 2333
E-mail : pkurt@ybu.edu.tr

EDUCATION

Master Degree : Ankara Yıldırım Beyazıt University / Mechanical Engineering (2014- continued)
Supervisors: Sadettin Orhan, Cihangir Duran
Bachelor : Gazi University / Mechanical Engineering (2008-2012)

EXPERIENCE

Work Experience
2014 - Ankara Yıldırım Beyazıt University, Ankara, Research Assistant
2012-2014 Orbitek Orm. Ve Biyo-Tek Sis. Ltd. Şti, Ankara, Mechanical Engineer

Internships
2012 Türk Traktör-Ankara-Intern of Management
2011 TCDD-Ankara-Intern of Energy
2010 Ficosa A.Ş.-Bursa-Intern of Manufacturing

LANGUAGE

English – A (YDS-91.25)
QUALIFICATIONS

MS Office
AutoCAD
Solidwors
Matlab
Comsol

PROJECTS

Design, Production and Acoustic Analysis of Transducer, BAP Project, Researcher, (06/02/2016-…)

Otomatic Pruning Machine Design, Teknogirisim (Ministry of Science, Industry and Technology) Project

INTERNATIONAL CONFERENCES


TOPICS OF INTEREST

- Underwater Acoustics
- Mechanical Vibrations
- Mechanics of Materials
- Machinery Dynamics